Advanced Topics in Term Rewriting LVA 703610

http://cl-informatik.uibk.ac.at/teaching/ws06/attr/

Georg Moser

office hours: Tuesday, 16:00–18:00 (3M09)

Advanced Topics in Term Rewriting

G. Moser

Bottom-Up Tree Automata

Tree Automata

- ightharpoonup tree automaton is quadruple $\mathcal{A}=(\mathcal{F},Q,Q_f,\Delta)$ with
 - \bigcirc \mathcal{F}
- signature
- states
- ③ $Q_f \subseteq Q$ final states
- transition rules

$$f(\alpha_1, \dots, \alpha_n) \rightarrow \beta$$
 $\alpha \rightarrow \beta$ epsilon transition

 \rightarrow language accepted by A:

$$L(\mathcal{A}) = \{ t \in \mathcal{T}(\mathcal{F}) \mid \exists \alpha \in Q_f \colon t \xrightarrow{*}_{\Delta} \alpha \}$$

 $ightharpoonup L \subseteq \mathcal{T}(\mathcal{F})$ is regular if $L = L(\mathcal{A})$ for some tree automaton \mathcal{A}

tree automaton ${\cal A}$

- ightharpoonup states α $\underline{\beta}$ final

$$\begin{array}{lll} f(f(a)) \in \mathit{L}(\mathcal{A}) \colon & f(f(a)) \to f(f(\alpha)) \to f(\beta) \to f(\alpha) \to \beta \\ \\ g(a) \notin \mathit{L}(\mathcal{A}) \colon & g(a) \to g(\alpha) \to \alpha \end{array}$$

$$\begin{array}{lll} \mathit{L}(\mathcal{A}) = \{ t \mid \mathsf{root}(t) = f \} \end{array}$$

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3

Bottom-Up Tree Automata

Deterministic Tree Automata

Closure Properties

Decidability

Examples of regular tree languages

- ⇒ set of all ground terms
- ⇒ set of well-typed terms with respect to order-sorted signature

sorts	<pre>nat < int bool</pre>	
signature	$\begin{array}{lll} \textbf{0}: & \textbf{nat} \\ \textbf{s}: & \textbf{nat} & \rightarrow & \textbf{nat} \\ \textbf{s}: & \textbf{int} & \rightarrow & \textbf{int} \\ \textbf{p}: & \textbf{int} & \rightarrow & \textbf{int} \end{array}$	$+:$ nat \times nat \rightarrow nat $+:$ int \times int \rightarrow int $\leqslant:$ int \times int \rightarrow bool
tree automaton	$egin{array}{lll} 0 & ightarrow & ext{nat} \ ext{s(nat)} & ightarrow & ext{nat} \ ext{s(int)} & ightarrow & ext{int} \ ext{p(int)} & ightarrow & ext{int} \end{array}$	$egin{array}{lll} +(\mathtt{nat},\mathtt{nat}) & ightarrow & \mathtt{nat} \ +(\mathtt{int},\mathtt{int}) & ightarrow & \mathtt{int} \ \leqslant (\mathtt{int},\mathtt{int}) & ightarrow & \mathtt{bool} \ & \mathtt{nat} & ightarrow & \mathtt{int} \end{array}$

Deterministic Tree Automata

Theorem

every regular language is accepted by deterministic completely defined tree automaton $\mathcal{A} = (\mathcal{F}, Q, Q_f, \Delta)$

- no epsilon transitions
- no different transition rules with same left-hand sides
- $\forall f \in \mathcal{F} \ \forall \alpha_1, \ldots, \alpha_n \in Q \ \exists f(\alpha_1, \ldots, \alpha_n) \to \beta \in \Delta$

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Proof by Example

$$egin{array}{cccc} & \mathsf{a} &
ightarrow & lpha \ \mathsf{f}(lpha) &
ightarrow & eta \ \mathsf{g}(lpha) &
ightarrow & lpha \ eta &
ightarrow & lpha \ eta &
ightarrow & lpha \end{array}$$

$$\begin{array}{cccc} \mathsf{a} & \to & \alpha \\ \mathsf{f}(\alpha) & \to & \underline{\beta} \\ \mathsf{g}(\alpha) & \to & \alpha \end{array} \qquad \mathcal{A}_2$$

$$\mathsf{f}(\alpha) & \to & \alpha \end{array}$$

① remove epsilon transitions

$$\begin{array}{ccc} \mathsf{a} & \to & \{\alpha\} \\ \mathsf{f}(\varnothing) & \to & \varnothing \\ \mathsf{f}(\{\alpha\}) & \to & \{\alpha, \beta\} \\ \mathsf{f}(\underline{\{\beta\}}) & \to & \varnothing \\ \mathsf{f}(\{\alpha, \beta\}) & \to & \{\alpha, \beta\} \end{array}$$

$$g(\varnothing) \rightarrow \varnothing$$

$$g(\{\alpha\}) \rightarrow \{\alpha\} \quad \mathcal{A}_3$$

$$g(\underline{\{\beta\}}) \rightarrow \varnothing$$

$$g(\{\alpha, \beta\}) \rightarrow \{\alpha\}$$

- ② subset construction
- ③ remove inaccessible states (optional)

Closure Properties

Theorem

regular languages are effectively closed under union, intersection and difference

Proof

union

tree automaton $\mathcal{A}_1 = (\mathcal{F}, Q_1, Q_f^1, \Delta_1)$ tree automaton $\mathcal{A}_2 = (\mathcal{F}, Q_2, Q_f^2, \Delta_2)$

$$\mathsf{wlog}\ \mathit{Q}_1\cap \mathit{Q}_2=\varnothing$$

 $L(A_1) \cup L(A_2) = L(A)$ for tree automaton $A = (\mathcal{F}, Q, Q_f, \Delta)$ with

- $ightharpoonup Q_1 \cup Q_2$
- $ightharpoonup Q_f = Q_f^1 \cup Q_f^2$
- \rightarrow Δ = $\Delta_1 \cup \Delta_2$

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7

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Proof

intersection

tree automaton $\mathcal{A}_1 = (\mathcal{F}, Q_1, Q_f^1, \Delta_1)$ tree automaton $\mathcal{A}_2 = (\mathcal{F}, Q_2, Q_f^2, \Delta_2)$

 $L(\mathcal{A}_1) \cap L(\mathcal{A}_2) = L(\mathcal{A})$ for tree automaton $\mathcal{A} = (\mathcal{F}, Q, Q_f, \Delta)$ with

- $ightharpoonup Q_1 \times Q_2$
- $ightharpoonup Q_f = Q_f^1 \times Q_f^2$
- **→** ∆:

$$f([\alpha_{1}, \beta_{1}], \dots, [\alpha_{n}, \beta_{n}]) \to [\alpha, \beta] \quad \forall \ f(\alpha_{1}, \dots, \alpha_{n}) \to \alpha \in \Delta_{1}$$

$$\forall \ f(\beta_{1}, \dots, \beta_{n}) \to \beta \in \Delta_{2}$$

$$[\alpha, \beta] \to [\alpha', \beta] \quad \forall \ \alpha \to \alpha' \in \Delta_{1}$$

$$[\alpha, \beta] \to [\alpha, \beta'] \quad \forall \ \beta \to \beta' \in \Delta_{2}$$

Proof

difference

tree automaton $\mathcal{A}_1 = (\mathcal{F}, Q_1, Q_f^1, \Delta_1)$ tree automaton $\mathcal{A}_2 = (\mathcal{F}, Q_2, Q_f^2, \Delta_2)$

deterministic completely defined

 $L(\mathcal{A}_1) \setminus L(\mathcal{A}_2) = L(\mathcal{A})$ for tree automaton $\mathcal{A} = (\mathcal{F}, Q, Q_f, \Delta)$ with

- $ightharpoonup Q_1 \times Q_2$
- $ightharpoonup Q_f = Q_f^1 \times (Q_2 \setminus Q_f^2)$
- \rightarrow \triangle

$$f([\alpha_{1}, \beta_{1}], \dots, [\alpha_{n}, \beta_{n}]) \to [\alpha, \beta] \quad \forall \ f(\alpha_{1}, \dots, \alpha_{n}) \to \alpha \in \Delta_{1}$$
$$\forall \ f(\beta_{1}, \dots, \beta_{n}) \to \beta \in \Delta_{2}$$
$$[\alpha, \beta] \to [\alpha', \beta] \quad \forall \ \alpha \to \alpha' \in \Delta_{1}$$

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9

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Decidability

Decidability

Theorem

ightharpoonup membership instance: tree automaton $\mathcal A$ and term t

question: $t \in L(A)$?

ightharpoonup emptiness instance: tree automaton $\mathcal A$

question: $L(A) = \emptyset$?

ightharpoonup finiteness instance: tree automaton $\mathcal A$

question: L(A) is finite?

 \rightarrow inclusion instance: tree automata A_1 and A_2

question: $L(A_1) \subseteq L(A_2)$?

are decidable problems