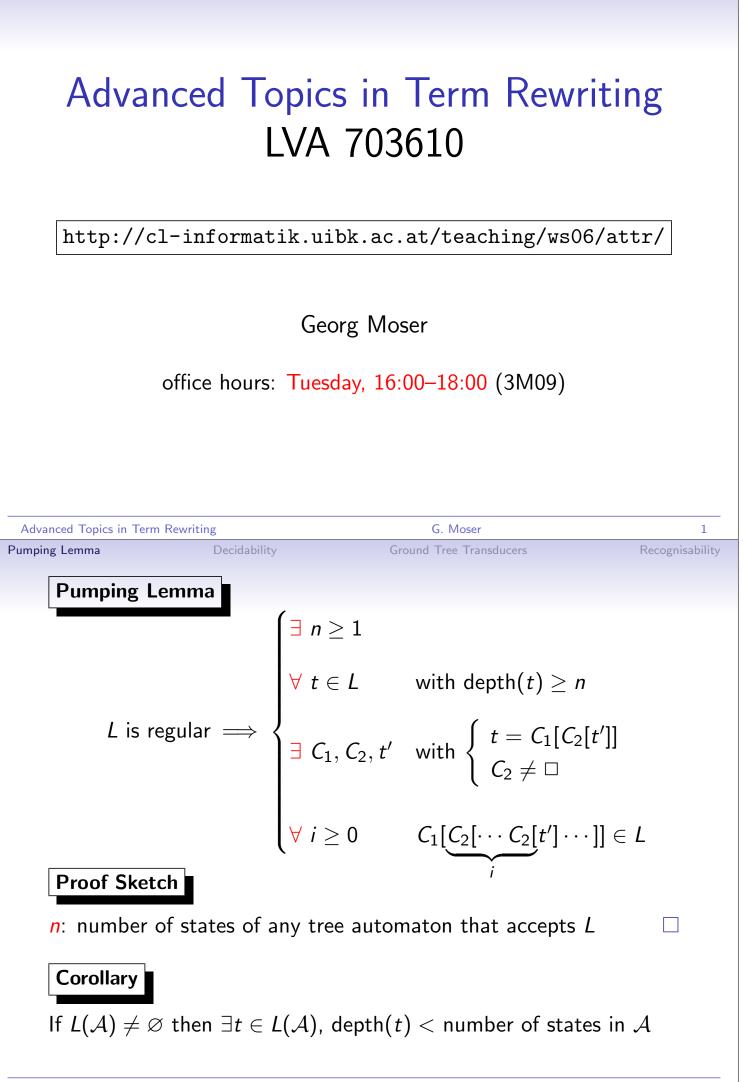
Recognisability



Advanced Topics in Term Rewriting

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Pumping Lemma	Decidability	Ground Tree Transducers	Recognisability	
Decidability				
Theorem				
➡ member	ship			
instance				
•	: $t \in L(\mathcal{A})$ ?			
emptine instance		$r \exists \ t \in L(\mathcal{A})$ : depth $(t) <  Q $ on $\mathcal{A}$		
	: $L(\mathcal{A}) = \emptyset$ ?			
➡ finitenes	s ↔ -	$r \exists t \in L(\mathcal{A}) \colon  \mathcal{Q}  \leqslant depth(t)$	() < 2 Q	
	instance: tree automaton $\mathcal{A}$			
question inclusion	: $L(\mathcal{A})$ is finite	<u>'</u>		
instance				
question	: $L(\mathcal{A}_1) \subseteq L(\mathcal{A})$	<sub>2</sub> )?		
are decidable	problems			
Advanced Topics in Term F Pumping Lemma	Rewriting Decidability	G. Moser Ground Tree Transducers	3 Recognisability	
Words and Terms				
	vorus			
Definition				
1 vield( $L$ )	$=$ { yield(t)   t $\in$	: <i>L</i> }		
		,		
<b>2</b> yield $(t)$	$= \begin{cases} c \\ vield(t_1) \cdots v \end{cases}$	if $t = a$ yield $(t_n)$ if $t = f(t_1, \dots, t_n)$	)	
	() ()		)	
Theorem				
		6		
	t-free word gram	imar G regular tree language		
	r tree language <i>L</i>			
yield(L) is context-free word language				



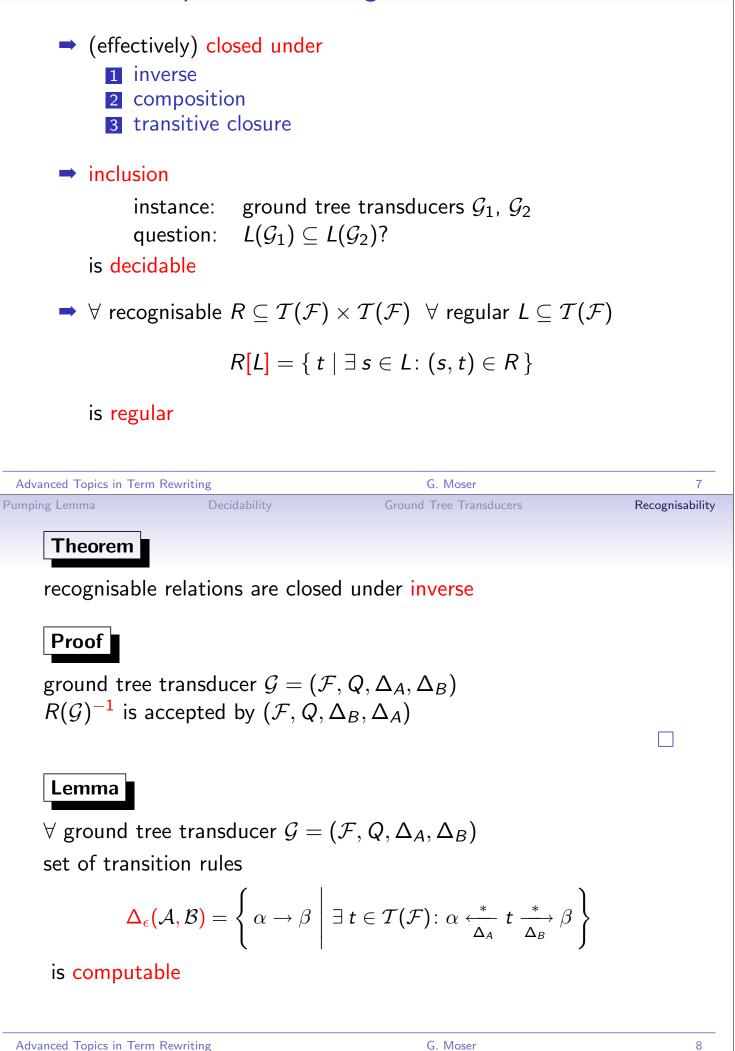
rightarrow ground tree transducer is pair  $\mathcal{G} = (\mathcal{A}, \mathcal{B})$  of tree automata  $\mathcal{A} = (\mathcal{F}, Q_A, -, \Delta_A)$  $\mathcal{B} = (\mathcal{F}, Q_B, -, \Delta_B)$  $\rightarrow$  relation accepted by  $\mathcal{G}$ :  $R(\mathcal{G}) = \{ (s, t) \mid s \xrightarrow{*}_{\Delta_A} \cdot \xleftarrow{*}_{\Delta_B} t \}$  $\Rightarrow$   $R \subseteq \mathcal{T}(\mathcal{F}) \times \mathcal{T}(\mathcal{F})$  is recognisable if  $R = R(\mathcal{G})$  for some ground tree transducer  $\mathcal{G}$ G. Moser Advanced Topics in Term Rewriting Pumping Lemma Ground Tree Transducers Recognisability Theorem  $\#_{\mathcal{R}}$  is recognisable for finite left-linear right-ground  $\mathcal{R}$ Proof by Example  $0 + \gamma \rightarrow S(0)$  $S(x) + y \rightarrow S(0+0)$  $\mathcal{R}$  $0 \times y \rightarrow 0$  $S(x) \times y \rightarrow 0 + 0$  $\#_{\mathcal{R}} = R(\mathcal{G}) \qquad \mathcal{G} = (\mathcal{A}, \mathcal{B})$ 0  $0 \rightarrow 0$  $\rightarrow$  \*  $S(*) \rightarrow *$  $0+0 \rightarrow 0+0$  $S(*) \rightarrow S(*) S(*) \times * \rightarrow 4$  $4 \leftarrow 0 + 0$ 

Advanced Topics in Term Rewriting

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Decidability **Pumping Lemma** Ground Tree Transducers Theorem recognisable relations are closed under composition Proof ground tree transducer  $\mathcal{G}_1 = (\mathcal{F}, Q_1, \Delta_{A_1}, \Delta_{B_1})$ ground tree transducer  $\mathcal{G}_2 = (\mathcal{F}, Q_2, \Delta_{A_2}, \Delta_{B_2})$ wlog  $Q_1 \cap Q_2 = \emptyset$  $R(\mathcal{G}_1) \cdot R(\mathcal{G}_2)$  is accepted by  $\mathcal{G} = (\mathcal{F}, Q, \Delta_A, \Delta_B)$  with  $\Rightarrow Q = Q_1 \cup Q_2$  $\Rightarrow \Delta_{\mathcal{A}} = \Delta_{\mathcal{A}_1} \cup \Delta_{\mathcal{A}_2} \cup \Delta_{\epsilon}(\mathcal{B}_1, \mathcal{A}_2)$  $\Rightarrow \Delta_B = \Delta_{B_1} \cup \Delta_{B_2} \cup \Delta_{\epsilon}(\mathcal{A}_2, \mathcal{B}_1)$ Advanced Topics in Term Rewriting G. Moser 9 Ground Tree Transducers Recognisability **Pumping Lemma** Theorem recognisable relations are closed under transitive closure Proof ground tree transducer  $\mathcal{G} = (\mathcal{F}, Q, \Delta_A, \Delta_B)$ define  $\mathcal{G}_i = (\mathcal{F}, Q, \Delta_{A_i}, \Delta_{B_i})$  with  $R(\mathcal{G})^+$  is accepted by  $\mathcal{G}_n$  for *n* with  $\mathcal{G}_n = \mathcal{G}_{n+1}$ 

Decidability

Recognisability

