Functional Programming | WS 2007/2008 | LVA 703018

Solutions

1.

$$
\begin{aligned}
\texttt{drop 2 [5; 7; 2; 4]} &\rightarrow^* \texttt{if 2 <= 0 then [5; 7; 2; 4] else drop (2 - 1) [7; 2; 4]} \\
&\rightarrow^* \texttt{if false then [5; 7; 2; 4] else drop (2 - 1) [7; 2; 4]} \\
&\rightarrow^* \texttt{drop (2 - 1) [7; 2; 4]} \\
&\rightarrow^* \texttt{drop 1 [7; 2; 4]} \\
&\rightarrow^* \texttt{if 1 <= 0 then [7; 2; 4] else drop (1 - 1) [2; 4]} \\
&\rightarrow^* \texttt{if false then [7; 2; 4] else drop (1 - 1) [2; 4]} \\
&\rightarrow^* \texttt{drop (1 - 1) [2; 4]} \\
&\rightarrow^* \texttt{drop 0 [2; 4]} \\
&\rightarrow^* \texttt{if 0 <= 0 then [2; 4] else drop (0 - 1) [4]} \\
&\rightarrow^* \texttt{if true then [2; 4] else drop (0 - 1) [4]} \\
&\rightarrow^* \texttt{[2; 4]}
\end{aligned}
$$

2.

```
let times xs = fold ( * ) 1 xs;;
```

3.

```
let rec sum_nodes = function
  | Empty -> 0
  | Node (l, a, r) -> a + (sum_nodes l) + (sum_nodes r)
;;
let rec sum_tree = function
  | Empty -> Empty
  | (Node (l, a, r)) as t -> Node (sum_tree l, sum_nodes t, sum_tree r)
;;
```

4. (a)

$$\begin{aligned}\mathcal{S}\mathsf{ub}(t) = \{&\lambda xw.(\lambda xz.y\ z)\ w\ z,\\ &\lambda w.(\lambda xz.y\ z)\ w\ z,\\ &(\lambda xz.y\ z)\ w\ z,\\ &(\lambda xz.y\ z)\ w,\\ &\lambda xz.y\ z,\\ &\lambda z.y\ z,\\ &y\ z,\\ &w,\\ &y,\\ &z\\ \}&\end{aligned}$$

(b)

$$\begin{aligned}\mathcal{V}\mathsf{ar}(t) &= \{w, x, y, z\}\\ \mathcal{BV}\mathsf{ar}(t) &= \{w, x, z\}\\ \mathcal{FV}\mathsf{ar}(t) &= \{y, z\}\end{aligned}$$

(c) No, since following β-step is possible:

$$\lambda xw.\underline{(\lambda xz.y\ z)\ w}\ z \to_\beta \lambda xw.(\lambda z'.y\ z')\ z$$