University of Innsbruck
Test 2A

Institute of Computer Science
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Functional Programming | WS 2007/2008 | LVA 703018

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This test consists of three exercises. *Explain your answers.* The available points for each item are written in the margin.

[7] 1. Consider the λ-term $t = (\lambda f.(\lambda x.f\ (x\ x))\ (\lambda x.f\ (x\ x)))\ (\lambda yz.z)$. Use the leftmost outermost reduction strategy to reduce t as far as possible.

[8] 2. Consider the functions:

```
let rec sum = function
  | [] -> 0
  | x :: xs -> x + sum xs
;;
```

and

```
let rec fold f b = function
  | [] -> b
  | x :: xs -> f x (fold f b xs)
;;
```

Prove by induction over xs that for all integer lists xs it holds that

$$\mathsf{sum}\ xs = \mathsf{fold}\ (+)\ 0\ xs.$$

3. For each of the following functions, decide whether it is tail recursive. If yes, justify your answer. Give a tail recursive implementation otherwise.

[5] (a) Consider the function sum from Exercise 2.

[5] (b) Consider the function f

```
let rec f = function
  | [] -> []
  | x :: xs -> f xs
;;
```