Name: ________________________ Points: ____

Matr.-Nr.: __________________

1 Formalisation (15 pts)

Consider the following statements:

① If a car has a diesel engine and is refueled with gas then it gets defect.

② A mechanic cannot repair a defect car if she/he does not have the necessary tool for that car.

③ There is a blue car which is refueled with gas.

④ All defect cars are blue and all blue cars have diesel engine.

⑤ For every car with diesel engine there is a mechanic which has the necessary tool for that car.

⑥ There is a defect car which cannot be repaired by any mechanic.

a) Give a first-order formula that formalises the statements above. Use therefore the following constants, functions and predicates: [5]

- constants: *diesel*, *gas* and functions: $\mathit{tool}(x)$
  [first letter suffices, hence you can use $d, g, t(x)$]
- predicates: $\mathit{car}(x)$, $\mathit{engine}(x, y)$, $\mathit{refuel}(x, y)$, $\mathit{mechanic}(x)$, $\mathit{repair}(x, y)$, $\mathit{defect}(x)$, $\mathit{has}(x, y)$, $\mathit{blue}(x)$ [first 3 letters suffice to identify the predicate]

Note that the function $\mathit{tool}(x)$ returns the tool needed to repair car x. Further, the predicate $\mathit{engine}(x, y)$ has to be interpreted as "x has a y engine", the predicate $\mathit{refuel}(x, y)$ as "x is refueled with y", the predicate $\mathit{repair}(x, y)$ as "x can repair y", the predicate $\mathit{has}(x, y)$ as "x has y".

b) Is your formalization satisfiable? Explain your answer. [5]

c) Change ② so that the answer to b) changes and shortly explain why it changes. [5]

2 Resolution (10 pts)

a) Decide for the following set of clauses

$$\{\neg p, q, \neg r\} \quad \{q, \neg r, \neg s\} \quad \{p, q, \neg s\} \quad \{\neg p, \neg q, \neg s\}$$
$$\{p, \neg q, \neg s\} \quad \{\neg q, r, s\} \quad \{q, r, s\} \quad \{\neg r, s\}$$

whether the empty clause can be derived or not. If the empty clause can be derived, prove it by resolution. Otherwise give a satisfying truth assignment (valuation). [4]

b) Prove by resolution that:

$$\begin{aligned}\forall x, y\,\big(P(x, f(x), f(y)) \to Q(y, x)\big) \quad &\wedge \\ \forall x, y\,\big(Q(x, y) \to Q(g(x), f(y))\big) \quad &\wedge \\ \forall x, y, z\, P(x, y, f(g(z))) \quad &\vdash \quad \forall x\, Q(g(g(g(x))), f(f(x)))\end{aligned}$$

Display the resolution tree and give all substitutions employed in its construction. [6]

3 Resolution from semantic tree (5pt)

Consider the following set of clauses

$$\{p, \neg q, r\} \quad \{\neg p, \neg q\} \quad \{p, q, r\} \quad \{q\} \quad \{p, \neg q, \neg r\}$$

a) Give the semantic tree. Mark the failure nodes and indicate their associated clauses. [2.5]

b) Construct a resolution derivation of the empty clause in the following way:

① In each step choose an inference node from the semantic tree from a)

② Resolve associated clauses of the two children of the inference node.

③ Mark new failure nodes in the semantic tree and indicate associated clauses.

④ Repeat until no more inference node. [2.5]

4 Theoretical Questions (10 pts)

Give the (correct) answers to the following questions:

a) Explain the difference between $\models$ and $\vdash$ with respect to propositional logic.
How are $\models$ and $\vdash$ *related* to each other? [3]

b) Is it *decidable* whether a propositional formula is valid or not? Cross the right answer!

☐ Yes ☐ No ☐ Unknown [1]

c) Give the definition of *Herbrand expansion*, state and prove its associated lemma. Outline the *Ground-Resolution* procedure. Give the proof of *completeness* of this procedure. In particular, name and state the main theorems used in this short proof (no need to prove the main theorems). [5]

d) Is it *decidable* whether a first-order formula is valid or not? Cross the right answer!

☐ Yes ☐ No ☐ Unknown [1]