Name: ________________________ Points: ____

Matr.-Nr.: _________________

1 First-order logic (15 pts)

Consider the following natural-language sentences:

① If a dragon is blue then its father or mother is blue.

② Red dragons can spit fire.

③ A dragon can perform magic if all its relatives can perform magic.

④ There are blue dragons which cannot spit fire.

⑤ Saphira is a blue dragon who cannot perform magic.

It is tacitly assumed that the father and the mother of a dragon are dragons as well.

a) Give a first-order formula that formalises the sentences above. Use therefore the following constants, functions and predicates: [5]

- constants: *Saphira*
- functions: $\mathit{father}(x)$, $\mathit{mother}(x)$
- predicates: $\mathit{spit\text{-}fire}(x)$, $\mathit{dragon}(x)$, $\mathit{blue}(x)$, $\mathit{red}(x)$, $\mathit{magic}(x)$, $\mathit{relative}(x, y)$

Note that the predicate $\mathit{magic}(x)$ has to be interpreted as "x performs magic", the predicate $\mathit{relative}(x, y)$ as "x is a relative of y".

b) Show that the complete formalization is satisfiable. [10]

2 First-order Resolution (5 pts)

Is the sentence

$$\exists x \exists y \big([P(b,x) \wedge Q(f(x),y)] \vee [\neg P(a,x) \wedge \neg Q(x,b)] \vee [\neg P(b,x) \wedge Q(f(x),b)] \vee [P(x,f(a)) \wedge \neg Q(f(x),y)] \big)$$

valid or not? If it is valid, display the resolution tree and give all substitutions employed in its construction. Otherwise give a satisfying interpretation for its negation. [5]

3 Expressiveness of first-order logic (5 pts)

State the Compactness Theorem and use it to prove the following:
Consider a signature which contains a binary predicate symbol R. There is no first-order sentence ϕ such that for every interpretation $\mathcal{M}$

$$\mathcal{M} \models \phi \quad \texttt{iff} \quad R^{\mathcal{M}} \text{ has at least one cycle}$$

A cycle of length n of $R^{\mathcal{M}}$ is a sequence $a_1, a_2, \ldots, a_n$ of elements of the universe of $\mathcal{M}$ for which (a_1, a_2), (a_2, a_3), $\ldots$, (a_{n-1}, a_n), $(a_n, a_1) \in R^{\mathcal{M}}$. [5]

4 Theoretical Questions - Propositional Logic (15 pts)

a) Give a Natural Deduction proof system for Propositional Logic. [3]

b) Give the (boolean) semantics of Propositional Logic. [3]

c) Give the definitions of the following elementary notions related to Resolution: *Literal*, *Conjunctive Normal Form (CNF)*, *Clause*, *Clausal form*, *Clashing Clauses* and their *Resolvent*. [HINT: brief (but complete) descriptions suffice.] [3]

d) Give the Propositional Resolution Algorithm. [HINT: briefly (but complete).] [3]

e) State the soundness and completeness theorems for Propositional Resolution. [3]