University of Innsbruck | **Institute of Computer Science**
3rd Exam | **May 14, 2009**

Functional Programming | **WS 2008/2009** | **LVA 703017**

Name: | **Matr.Nr.:**

1. Consider the lambda-term $t = (\lambda x\ y.x\ (x\ y))\ (\lambda x.s\ x)\ z$.

[10] (a) Reduce t stepwise to normal form, using the leftmost innermost strategy.

[10] (b) Reduce t stepwise to normal form, using the leftmost outermost strategy.

[20] **2.** Consider the type

```
type nat = Z | S of nat
```

together with the functions

```
let rec ( + ) x y = match x with Z -> y | S x -> S(x + y)
let rec ( * ) x y = match x with Z -> Z | S x -> y + x * y
```

Prove by induction that $a * (b + c) = a * b + a * c$ for all `nats` a, b, and c. You may use associativity and commutativity of `+`, i.e.,

$$a + (b + c) = (a + b) + c \qquad (\star)$$
$$a + b = b + a \qquad (\star\star)$$

for all `nats` a, b, and c.

3. Consider the OCaml function `initial`, defined by:

```
let rec initial = function x::y::xs -> x :: initial(y::xs) | _ -> []
```

[10] (a) Implement a tail-recursive variant of `initial`. You may assume to have a tail-recursive function `rev` (reversing a list) at hand.

[10] (b) Implement the function `even_odd` that splits a list into two lists, where the first contains all elements having even indices and the second all those with odd indices.

4. Consider the λ-term $t = (\lambda x\ y.x\ (x\ y))\ (\lambda x.s\ x)\ z$.

[5] (a) Is t in normal form? (Justify your answer.)

[5] (b) Give the set $\mathcal{FV}\mathsf{ar}(t)$ of free variables of t.

[5] (c) Give the set $\mathcal{BV}\mathsf{ar}(t)$ of bound variables of t.

[5] (d) Give the set $\mathcal{S}\mathsf{ub}(t)$ of all subterms of t.

[10] **5.** (a) Transform the type inference problem $E \triangleright (\lambda x\ y.x\ (x\ y))\ \mathsf{s}\ \mathsf{0} : \alpha_0$, using the environment $E = \{\mathsf{s} : \mathsf{int} \to \mathsf{int}, \mathsf{0} : \mathsf{int}\}$, into a unification problem.

[10] (b) Solve the following unification problem (if possible).

$$\begin{aligned}
&\alpha_3 \approx \alpha_7 \to \alpha_6 &\qquad& \alpha_2 \to \alpha_1 \to \alpha_0 \approx \alpha_3 \to \alpha_4\\
&\alpha_3 \approx \alpha_8 \to \alpha_7 && \mathsf{int} \to \mathsf{int} \approx \alpha_2\\
&\alpha_5 \approx \alpha_8 && \mathsf{int} \approx \alpha_1\\
&\alpha_4 \approx \alpha_5 \to \alpha_6 &&
\end{aligned}$$