Functional Programming
Exercises Week 7
(for November 21, 2008)

1. Read Section 5.5 and Chapter 6 of the lecture notes.

2. Reduce the λ-term $\mathsf{hd}\ (\mathsf{cons}\ \bar{1}\ \mathsf{nil})$ to WHNF using the leftmost outermost strategy.

3. Consider the infinite list of natural numbers `nats`, defined by

```
let rec from n = n :: from(n+1)
let nats       = from 0
```

Give the computation steps of `hd nats` using call-by-name. What happens using call-by-value?

4. Use the module `Lambda` from the archive of week 6 to reduce the term of Exercise 2 to WHNF.

5. Prove the following equation by induction over natural numbers:

$$x^{m+n} = x^m \cdot x^n$$

for all natural numbers m, n, and x.

6. Prove the following equation by structural induction over lists:

$$\texttt{rev'}(xs \,\texttt{@}\, ys) = \texttt{rev'}\ ys \,\texttt{@}\, \texttt{rev'}\ xs$$

where `rev'` and `@` are defined by

```
let rec (@) xs ys = match xs with []     -> ys
                                | x::xs -> x::(xs@ys)

let rec rev' = function []     -> []
                      | x::xs -> rev' xs @ [x]
```

and you may use the equations

$$xs \,\texttt{@}\, \texttt{[]} = xs \qquad (\star)$$

$$(xs \,\texttt{@}\, ys) \,\texttt{@}\, zs = xs \,\texttt{@}\, (ys \,\texttt{@}\, zs) \qquad (\star\star)$$

(the corresponding proofs can be found in the lecture notes).

Remark: For the induction proofs, state the property you want to prove and the IH you are using.