First Exam
Game Theory, LVA 703501

January 22, 2009

Name: **Studentnumber**:

The exam consists of 5 exercises with a total of 50 points.

1	2	3	4	5	Sum

1. Let $X \subseteq \mathbb{R}$ and X be finite with $x \in X$ a prize that amount to € x. Consider the following definition of $f \succcurlyeq_T g$:

$$\min_{s \in T} \sum_{x \in X} x \cdot f(x|s) \geqslant \min_{s \in T} \sum_{x \in X} x \cdot g(x|s) \ .$$

   a) Give an informal explanation of the relation $f \succcurlyeq_T g$. (2 pts)
   b) Does this definition of $\succcurlyeq_T$ violate any of the axioms on decision theory? (4 pts)
   c) Give an example of a preference (perhaps different from above) such that at least one axiom is violated. (3 pts)

2. Consider the following voting mechanism: Three committee members decide (vote) each secretly on an option α, β, γ. The the votes are counted. If any options gets two votes, then this option is the outcome. Otherwise player 1 (the chairperson) decides. The payoffs are as follows: If option α is voted, player 1 gets € 8 and player 3 € 4, for option β player 1 gets € 4 and player 2 gets € 8, and for option γ, player 2 gets € 4 and player 3 € 8. If a player is not metioned in this list, she gets nothing.

   a) Express the game in extensive form. (3 pts)
   b) Transform the game to reduced strategic form. (4 pts)
   c) Formalise the following assertion for games in extensive form as concrete as possible: *Whenever a player moves, she remembers all the information she knew earlier..* (4 pts)

3. Consider the following two games:

	P_2	
P_1	C	S
C	$-100,-100$	$1,0$
S	$0,1$	$0,0$

	Q_2	
Q_1	M	F
Rr	$0,0$	$1,-1$
Rp	$0.5,-0.5$	$0,0$
Pr	$-0.5,0.5$	$1,-1$
Pp	$0,0$	$0,0$

   a) Compute all Nash equilibria of the game Γ_1 to the left. (4 pts)
   b) Find all strongly dominated strategies of the game Γ_2 to the right. And define the fully reduced normal representation of Γ_2. (4 pts)
   c) Compute all Nash equilibria of Γ_2. (4 pts)

4. Answer the following question as completely as possible: *Why is every pure strategy equilibrium found by the Lemke-Howson Algorithm?*. (8 pts)

5. Determine whether the statements on the answer sheet are true or false. Every correct answer is worth 1 points (and every wrong -1 points). (10 pts)

statement	yes	no
To assert a player is intelligent, means the player makes decisions consistently in pursuit of her own objective.	☐	☐
Let X be a finite set of decisions and Ω a finite set of states. For any decision $y \in X$ that is strongly dominated by a randomised strategy $\sigma \in \Delta(X)$ there exists a probability distribution $p \in \Delta(\Omega)$ such that y is an optimal decision.	☐	☐
A set of vectors S is convex if for any two vectors p, q also $\lambda p + (1-\lambda)q \in S$, where $\lambda \in [0,1]$.	☐	☐
A game with incomplete information is a game in extensive form such that no two nodes have the same information set.	☐	☐
Given a finite game Γ in strategic form, there exists at least one pure equilibrium.	☐	☐
In a Dutch auction the seller starts from a price of zero and continuously raises this price. The auction is over when the penultimate bidder leaves the auction and is won by the remaining bidder.	☐	☐
A game may have multiple equilibria, but at least one of the equlibria is efficient.	☐	☐
A two-person game is called nondegenerated if all randomised strategies σ whose support has cardinality k have at most k pure best responses.	☐	☐
For a Nash equilibrium (σ, ρ) of a nondegenerated two-person game, σ and ρ have support of equal size.	☐	☐
If we can show that NASH $\in$ P, then P = NP follows.	☐	☐