Second Exam
Game Theory, LVA 703501

March 20, 2009

Name: **Studentnumber**:

The exam consists of 5 exercises with a total of 50 points.

1	2	3	4	5	Sum

1. Consider the following four axioms on preferences of decision makers for lotteries f, g, and h and event S:
   (i) $f \succcurlyeq_S g$ or $g \succcurlyeq_S f$,
   (ii) if $e \succ_S f$ and $g \succcurlyeq_S h$, $\alpha \in (0,1]$ then $\alpha e + (1-\alpha)g \succ_S \alpha f + (1-\alpha)h$,
   (iii) $f \succcurlyeq_S g$ and $g \succcurlyeq_S h$ implies $f \succcurlyeq_S h$, and
   (iv) if $f \succ_S h$ and $0 \leqslant \beta < \alpha \leqslant 1$, then $\alpha f + (1-\alpha)h \succ_S \beta f + (1-\beta)h$.

   (Here α, β are reals.) Prove the following two properties.
   a) The axiom (iii) follows from the first two. (5 pts)
   b) The axiom (iv) follows from the first two. (5 pts)

2. Consider the following voting mechanism: Three committee members decide (vote) each secretly on an option α, β, γ. Then the votes are counted. If any options gets two votes, then this option is the outcome. The payoffs are as follows: If option α is voted, player 1 gets € 4 and player 3 € 8, for option β player 1 gets € 5 and player 2 gets € 6, and for option γ, player 2 gets € 6 and player 3 € 5. If a player is not metioned in this list, she gets nothing.
   a) As defined the game is not well-defined. Introduce suitable rule(s) so that the game becomes well-defined. (2 pts)
   b) Express the game in extensive form. (4 pts)
   c) Transform the game to reduced strategic form. (4 pts)

3. Consider the following two games:

	P_2	
P_1	x_2	y_2
x_1	$2,1$	$1,2$
y_1	$1,5$	$2,1$

	Q_2	
Q_1	x_2	y_2
x_1	$3,7$	$6,6$
y_1	$2,2$	$7,3$

   a) Γ_1 and Γ_2 are both two-player games that have an odd number of equlibria. Give a concise explanation of this fact. (2 pts)
   b) Compute all Nash equilibria of the game Γ_1 to the left using the support enumeration technique. Provide sufficient evidence that all equlibira have been computed. (4 pts)
   c) Compute all Nash equilibria of Γ_2 to the right as above. (4 pts)

4. a) Define the Lemke-Howson algorithm including all necessary assumptions for its invocation. (5 pts)
   b) Define the complexity class PPAD and indicate the connection to the LH algorithm. (5 pts)

5. Determine whether the statements on the answer sheet are true or false. Every correct answer is worth 1 points (and every wrong -1 points). (10 pts)

statement	**yes**	**no**
To assert a player is intelligent, means the player is as smart as the observer.	☐	☐
A randomised strategy σ is a best response to a strategy τ if at least one strategy in the support set of σ is a best responses to τ.	☐	☐
The fully reduced normal representation is derived from the normal representation by eliminating all strategies thar are (randomly) redundant in the normal representation.	☐	☐
A strategy for player i in the Bayesian game is a function from the types of player i into the set of actions (of player i).	☐	☐
Given a finite game Γ in extensive form, there exists at least one pure equilibrium.	☐	☐
In an English auction the seller starts from a price of zero and continuously raises this price. The auction is over when the penultimate bidder leaves the auction and is won by the remaining bidder.	☐	☐
Baysian Nash equilibria differs slightly from Nash equlibria, in particular Baysian Nash equilibria need not be best responses.	☐	☐
A polyhedron is a polytope that is bounded.	☐	☐
A mechanism is called ex-ante efficient, if it allocates the objects to the bidder with the highest valuation.	☐	☐
If $\mathsf{NP} = \mathsf{P}$, then also $\mathsf{PPAD} = \mathsf{P}$.	☐	☐