1. a) *Solution.* We assume axioms (i) and (ii) and the assertions $f \succcurlyeq_S g$ and $g \succcurlyeq_S h$. In order to derive a contradiction, we assume $f \not\succcurlyeq_S h$. By totality we get $h \succ_S f$. Now, we apply axiom (i) with respect to $h \succ_S f$, $f \succcurlyeq_S g$, and $\alpha = \frac{1}{2}$. We conclude:

$$\frac{1}{2}h + \frac{1}{2}f \succ_S \frac{1}{2}f + \frac{1}{2}g \ . \tag{1}$$

We apply axiom (ii) again, this time with respect to (1), $g \succcurlyeq_S h$, and $\alpha = \frac{2}{3}$. We obtain:

$$\frac{2}{3}\frac{1}{2}h + \frac{2}{3}\frac{1}{2}f + \frac{1}{3}g \succ_S \frac{2}{3}\frac{1}{2}f + \frac{2}{3}\frac{1}{2}g + \frac{1}{3}h \ . \tag{2}$$

It is easy to see that both sides of the inequality (2) are equal, hence we derive the sought contradiction. Thus axiom (iii) follows from the first two. ☐

b) *Solution.* We assume the existence of α, β with $0 \leqslant \beta < \alpha \leqslant 1$ such that $f \succ_S h$ and $\alpha f + (1-\alpha)h \not\succ_S \beta f + (1-\beta)h$. Totality yields:

$$\beta f + (1-\beta)h \succcurlyeq_S \alpha f + (1-\alpha)h \ . \tag{3}$$

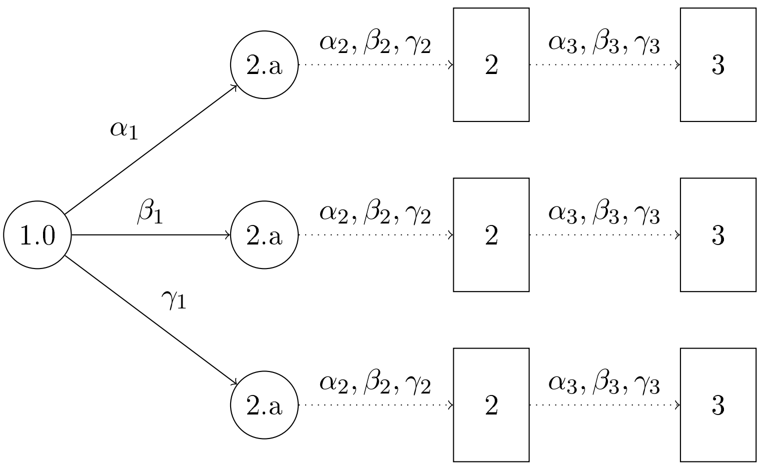
Applying axiom (ii) to $f \succ_S h$ and equation (3) for an arbitrary $\gamma > 0$ yields:

$$\gamma f + (1-\gamma)(\beta f + (1-\beta)h) \succ_S \gamma h + (1-\gamma)(\alpha f + (1-\alpha)h) \ . \tag{4}$$

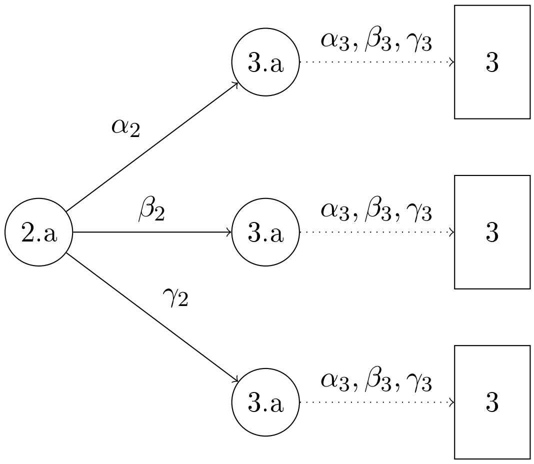
Comparing coefficients it is easy to see that for each α, β there exists a γ such that the sides of the inequality (4) are equal. We derive a contradiction. ☐

2. a) *Solution.* A default condition, if the vote is undecided is missing. We take the following assertion: In this case player 1 (the chairperson) decides. ☐

b) *Solution.* We write α_i, β_i, γ_i for the strategies of players $i = 1, 2, 3$. To clarify the presentation, we first draw the first level completely, but compress the subgames for player 2 and player 3.



In place of the boxes for player 2 in the above picture, the following subgame has to substituted:



The important thing in this substitution is that information states for players 2 (and 3) need to be copied, as player 2 (player 3) don't know the decisions of the other players. It is easy to see how to define the subgame that is substituted for the boxes for player 3 and the corresponding payoffs are computed: in case of an even vote, player 1 gets an additional decision node (e.g. marked $1.d$) where she always choses option β, as this gives the best payoff for her.

Finally, we have to merge payoff-equivalent strategies to obtain the reduced form. As any of three players has only three different strategies, and as the payoff for these strategies depends on the choices of the other players no pair of strategies is payoff-equivalent. □

c) *Solution.* The algorithm on page 48 in the book transforms the above extensive game Γ^e into its normal representation Γ. The reduced strategic form is (in principal) obtained by removing randomly redudant strategies, but this doesn't change Γ. We render below one row of the strategic form of Γ, where we fix the votes of player 1 and 2 to α.

	P_3		
P_1	α_3	β_3	γ_3
α_1	(4,0,8)	(4,0,8)	(4,0,8)

□

3. a) *Solution.* Both games are non-degenerated, hence the number of equlibria has to be odd. □

b) *Solution.* By the above property we know that there can only be an odd number of equilibria. Moreover it is easy to see that there is no pure Nash equilibrium.

First we consider the set of support to $\{x_1, y_1\} \times \{x_2, y_2\}$. Then we have the following equation:

$$u_2(\sigma_1, [x_2]) = \sigma_1(x_1) + 5\sigma_1(y_1) = 2\sigma_1(x_1) + \sigma_1(y_1) = u_2(\sigma_1, [y_2]) \ .$$

From this, we obtain $\sigma_1(y_1) = \frac{\sigma_1(x_1)}{4}$, which together with the constraint $\sigma_1(x_1) + \sigma_1(y_1) = 1$ yields:

$$\sigma_1(x_1) = \frac{4}{5} \qquad \sigma_1(y_1) = \frac{1}{5}$$

Now, we consider the utility of player 1:

$$u_1(\sigma_2, [x_1]) = 2\sigma_2(x_2) + \sigma_2(y_2) = \sigma_2(x_2) + 2\sigma_2(y_2) = u_1(\sigma_2, [y_1]) \ .$$

Again using the constraint $\sigma_2(x_2) + \sigma_2(y_2) = 1$, we compute:

$$\sigma_2(x_2) = \sigma_2(y_2) = \frac{1}{2} \ .$$

Hence $(\frac{4}{5}[x_1] + \frac{1}{5}[y_1], \frac{1}{2}[x_2] + \frac{1}{2}[y_2])$ is Nash equilibrium. Actually it is the only equilibrium.

For the other cases note that none of the supports $\{x_1, y_1\} \times \{x_2\}$, $\{x_1, y_1\} \times \{y_2\}$, $\{x_1\} \times \{x_2, y_2\}$, and $\{y_1\} \times \{x_2, y_2\}$ allows to satisfy all necessary constraints.

A simple calculation reveals that for a two person game we have to consider 9 distinct set of supports. Above we consider 5 of these and as the remaining support actually define pure equilibria, we are done. □

c) *Solution.* One proceeds as for Γ_1. □

4. a) *Solution.* See *Algorithmic Game Theory*, page 61ff for the definition. The important restriction is that the games are non-degenerated two-player games. □

   b) *Solution.* See *Algorithmic Game Theory*, page 36ff for a definition. The central connection is that the problem NASH is in PPAD, which one sees by the use of the LH algorithm. □

5.

statement	yes	no
To assert a player is intelligent, means the player is as smart as the observer.	✓	
A randomised strategy σ is a best response to a strategy τ if at least one strategy in the support set of σ is a best responses to τ.		✓
The fully reduced normal representation is derived from the normal representation by elliminating all strategies thar are (randomly) redundant in the normal representation.	✓	
A strategy for player i in the Bayesian game is a function from the types of player i into the set of actions (of player i).	✓	
Given a finite game Γ in extensive form, there exists at least one pure equilibrium.		✓
In an English auction the seller starts from a price of zero and continuously raises this price. The auction is over when the penultimate bidder leaves the auction and is won by the remaining bidder.	✓	
Baysian Nash equilibria differs slightly from Nash equlibria, in particular Baysian Nash equilibria need not be best responses.		✓
A polyhedron is a polytope that is bounded.		✓
A mechanism is called ex-ante efficient, if it allocates the objects to the bidder with the highest valuation.		✓
If $\mathsf{NP} = \mathsf{P}$, then also $\mathsf{PPAD} = \mathsf{P}$.	✓	