University of Innsbruck
Exam

Institute of Computer Science
28 January 2008

Introduction to Model Checking (VO) | WS 2007/2008 | LVA 703503

First name: _____

Last name: _____

Matriculation number: _____

- Write your name and matriculation number on every page.
- Please answer all exercises in a readable and precise way. Do *not* write with a pencil or a red pen. Please cross out solution attempts which are replaced by another solution.
- Cheating is not allowed. Everyone who is caught will fail the exam.
- Please do not remove the staples of the exam.

Exercise	Maximal points	Points
1(i)	12	
1(ii)	5	
2	20	
3	15	
4	18	
Σ	70	
Grade		

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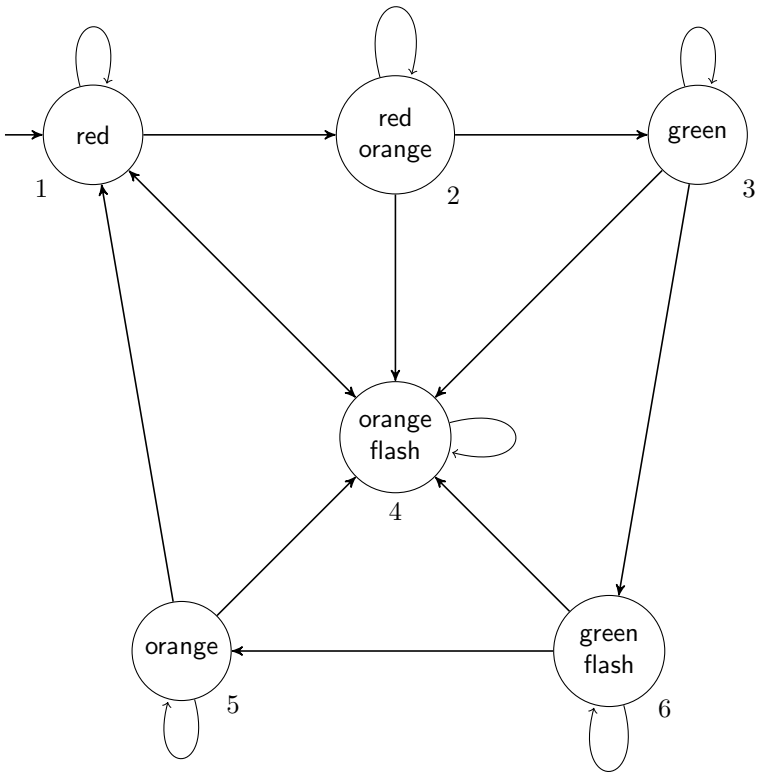
Exercise 1 (12 + 5 points)

Consider the following transition system TS of an Austrian traffic light using the atomic propositions {red, orange, green, flash}.



Here, state 4 represents a traffic light which is out-of-order.

(i) Consider the following property.

It is never allowed to switch from a red to a non-red state if the red state is not additionally orange. $(\star)$

One might formulate this property as the following CTL*-formula Φ.

$$\Phi = \neg\mathsf{red} \vee \mathsf{A}\,(\mathsf{X}\,\mathsf{red})\,\mathsf{U}\,(\mathsf{red} \wedge \mathsf{orange})$$

Compute $Sat(\Phi)$ by applying the CTL*-model checking algorithm. Here, the sets $Sat(\Psi)$ should be indicated for every state-subformula Ψ of Φ. Note that the subformula $\mathsf{red} \wedge \mathsf{orange}$ of Φ should be seen as a state-formula.

It is not necessary to perform the LTL-model checking explicitly, but write down the LTL-formula that is checked.

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(ii) By the results of part (i) one can easily see that $TS \not\models \Phi$. There are two reasons for this.

- Φ does not correspond to property ($\star$).
- Property ($\star$) does not take care of the possibility that the traffic light can become out-of-order.

Specify two formulas in the logic of your choice (LTL, CTL, CTL*) to fix these problems.

- One formula should exactly formalize ($\star$).
- The other formula should integrate the out-of-order possibility.

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Exercise 2 (20 points)

Consider the LTL-formula $\varphi = \mathsf{X}\, a \,\mathsf{U}\, b$ where a and b are atomic propositions. Using the construction to generate a GNBA from φ we obtain an NBA with the following states.

$$\begin{pmatrix}0\\0\\0\\0\end{pmatrix} \quad \begin{pmatrix}1\\0\\0\\0\end{pmatrix} \quad \begin{pmatrix}0\\1\\0\\0\end{pmatrix} \quad \begin{pmatrix}1\\1\\0\\0\end{pmatrix}$$

$$\begin{pmatrix}0\\0\\1\\0\end{pmatrix} \quad \begin{pmatrix}1\\0\\1\\0\end{pmatrix} \quad \begin{pmatrix}0\\1\\1\\0\end{pmatrix} \quad \begin{pmatrix}1\\1\\1\\0\end{pmatrix}$$

$$\begin{pmatrix}0\\0\\0\\1\end{pmatrix} \quad \begin{pmatrix}1\\0\\0\\1\end{pmatrix} \quad \begin{pmatrix}0\\1\\0\\1\end{pmatrix} \quad \begin{pmatrix}1\\1\\0\\1\end{pmatrix}$$

$$\begin{pmatrix}0\\0\\1\\1\end{pmatrix} \quad \begin{pmatrix}1\\0\\1\\1\end{pmatrix} \quad \begin{pmatrix}0\\1\\1\\1\end{pmatrix} \quad \begin{pmatrix}1\\1\\1\\1\end{pmatrix}$$

q_0

- Draw a circle around every final state.
- Draw all outgoing transitions of $\begin{pmatrix}0\\0\\1\\0\end{pmatrix}$ and all outgoing transitions of q_0.

Here, the vectors represent the formulas $\begin{pmatrix}a\\b\\\mathsf{X}\,a\\\mathsf{X}\,a\,\mathsf{U}\,b\end{pmatrix}$.

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Exercise 3 (15 points)

Construct the channel-system for the following nanoPromela program which models a sender which sends bits to a receiver via a lossy channel. The receiver computes the sum of the bits and the number of bits transferred.

```
------- SENDER PROCESS ------------
do :: true => if :: true => b = 0 :: true => b = 1 fi ; ch ! b od

------- RECEIVER PROCESS ----------
atomic { sum := 0; nr := 0 };
do :: true => ch ? value; nr = nr + 1; sum = sum + value od

------- LOSSY CHANNEL PROCESS -----
do :: true => ch ? dropped od
```

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Exercise 4 (18 points)

Each correct answer is worth two points. A wrong answer withdraws one point. Marking both "Yes" and "No" is a wrong answer only and does not result in $2 - 1 = 1$ points. It is not possible that the total score of this exercise is negative.

	Yes	No
The CTL formula $\mathsf{A}\,\mathsf{F}\,\mathsf{A}\,\mathsf{F}\,b$ is equivalent to the LTL formula $a\,\mathsf{U}\,\mathsf{F}\,b$.		
There is an LTL formula which is equivalent to $\neg\mathsf{E}\,a\,\mathsf{U}\,\mathsf{X}\,b$.		
The size of the transition system of a program graph is polynomial in the number of locations of the program graph.		
There exists a transition system, a state s, and a CTL*-state-formula Φ such that neither $s \models \Phi$ nor $s \models \neg\,\Phi$.		
Assume $P \neq NP$. The Hamiltonian Path Problem can be encoded in polynomial time as a CTL model checking problem.		
There exists an NBA $\mathcal{A}$ such that $\mathcal{L}(\mathcal{A}) \neq \mathcal{L}(\varphi)$ for every every LTL formula φ.		
Let channel c use hand-shaking. Then no state with an empty channel c can perform a transition which involves receiving a value along c. To be more precise, there is no state $\langle \ell_1, \ldots, \ell_i, \ldots, \ell_n, \eta, \xi \rangle$ where $\ell_i \xrightarrow{c?x}_i \ell_i'$ is a transition in the program graph and where $\xi(c) = \varepsilon$ such that a transition to a state of the form $\langle \ell_1', \ldots, \ell_i', \ldots, \ell_n', \eta', \xi' \rangle$ with $\ell_i \neq \ell_i'$ is possible.		
Given a DBA $\mathcal{A}_1$ and a GNBA $\mathcal{A}_2$, then $\overline{\mathcal{L}(\mathcal{A}_1)} \setminus \mathcal{L}(\mathcal{A}_2)$ is recognizable by an NBA.		
There is an N**F**A $\mathcal{A}_1$ and a GN**B**A $\mathcal{A}_2$ such that $\mathcal{L}(\mathcal{A}_1) = \mathcal{L}(\mathcal{A}_2)$.		