University of Innsbruck — Institute of Computer Science

2. Exam — 25 February 2008

Introduction to Model Checking (VO) — WS 2007/2008 — LVA 703503

First name: ______________________

Last name: ______________________

Matriculation number: ______________________

- Write your name and matriculation number on every page.
- Please answer all exercises in a readable and precise way. Do *not* write with a pencil or a red pen. Please cross out solution attempts which are replaced by another solution.
- Cheating is not allowed. Everyone who is caught will fail the exam.
- Please do not remove the staples of the exam.

Exercise	Maximal points	Points
1	17	
2	14	
3	17	
4	12	
Σ	60	
Grade		

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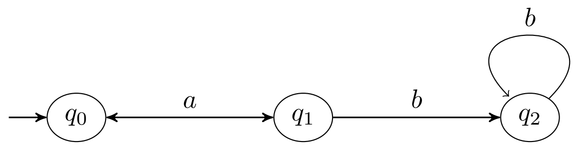
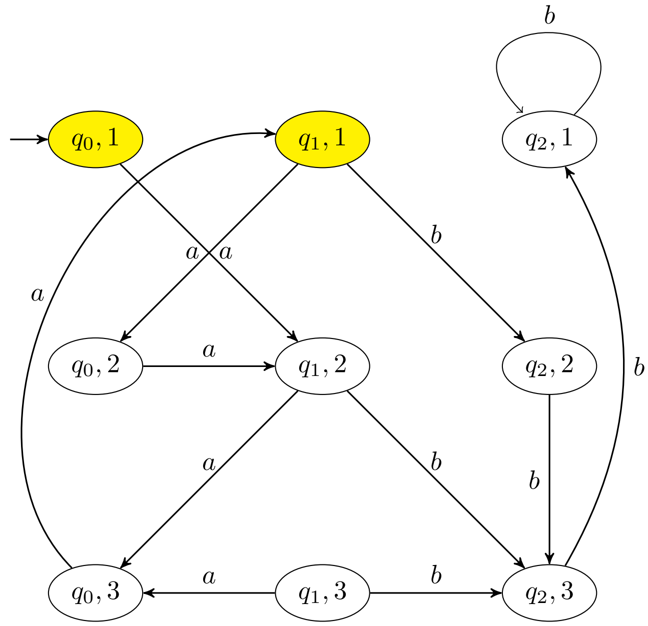
Exercise 1 (15 + 2 points)

Consider the GNBA $\mathcal{A} = (\{q_0, q_1, q_2\}, \{a, b\}, q_0, \delta, F_1, F_2, F_3)$ where $F_1 = \{q_0, q_1\}$, $F_2 = \{q_1, q_2\}$, and $F_3 = \{q_0, q_2\}$, and where δ is represented graphically.



- Construct the corresponding equivalent NBA.



Here, the yellow states mark the final states.

- Is $\mathcal{L}(\mathcal{A}) = \varnothing$? If not, then provide a word which is contained in $\mathcal{L}(\mathcal{A})$.

  $\mathcal{L}(\mathcal{A}) = \{a\,a\,a\,a\,\ldots\}$.

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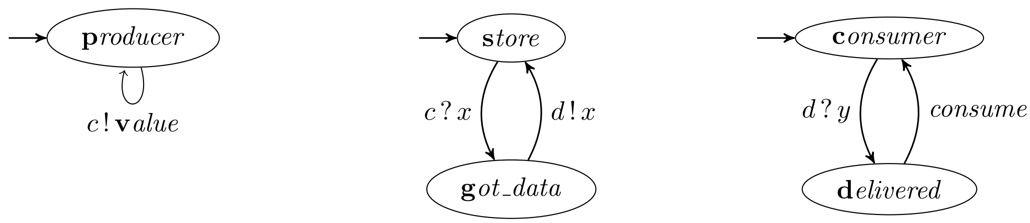
Exercise 2 (14 points)

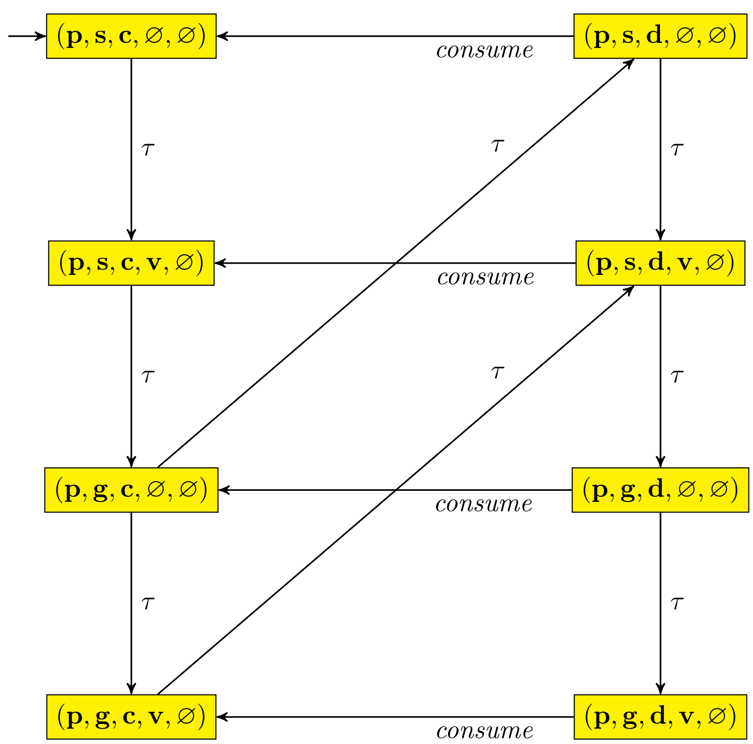
Consider the following channel system which transmits values from a producer via a store to a consumer.



We assume that the capacity of channel c is 1 and the capacity of channel d is 0. To construct the transition system for this channel system we will encounter states of the form

$$(\ell_1, \ell_2, \ell_3, \mathit{Eval}(c), \mathit{Eval}(d))$$

where we ignore the evaluation of variables since there is only one possible value. Here, ℓ_1, ℓ_2, and ℓ_3 are (the first letters of) the locations, i.e., $\ell_1 \in \{\mathbf{p}\}$, $\ell_2 \in \{\mathbf{s}, \mathbf{g}\}$, and $\ell_3 \in \{\mathbf{c}, \mathbf{d}\}$. Of course, $\mathbf{v}\mathit{alue}$ can be abbreviated by $\mathbf{v}$. Some initial part of the transition system is already depicted below. Draw the remaining parts.



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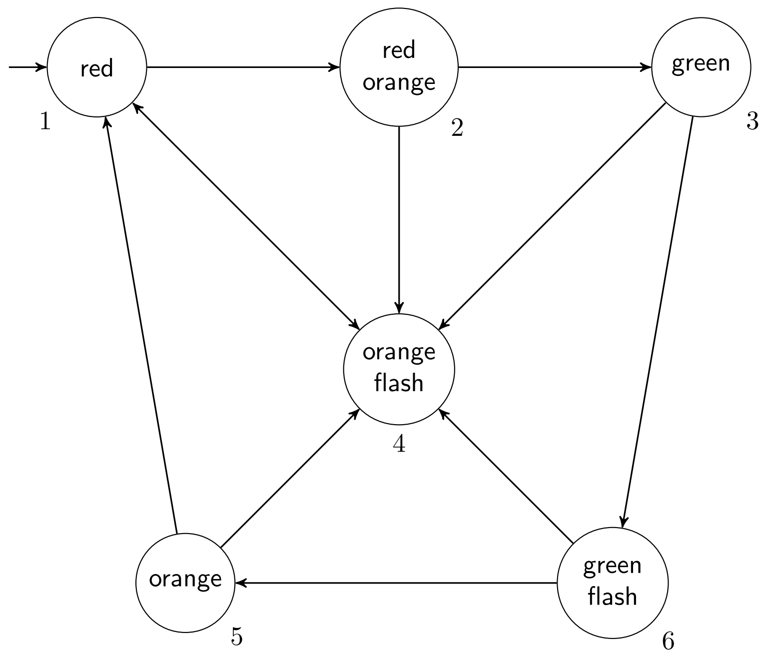
Exercise 3 (2 + 15 points)

Consider the following transition system TS of an Austrian traffic light using the atomic propositions {red, orange, green, flash}.



Consider the following CTL*-formula Φ.

$$\Phi = \mathsf{A}\,\mathsf{G}\,(\neg\mathsf{red} \vee \mathsf{E}\,(\mathsf{orange}\,\mathsf{U}\,\mathsf{X}\,\mathsf{red}))$$

Perform CTL*-model checking to decide whether the transition systems satisfies Φ.

(i) Compute a formula Φ' which is equivalent to Φ and does not contain E.

$$\Phi' = \mathsf{A}\,\mathsf{G}\,(\neg\mathsf{red} \vee \neg\mathsf{A}\,\neg(\mathsf{orange}\,\mathsf{U}\,\mathsf{X}\,\mathsf{red}))$$

(ii) Compute $Sat(\Psi)$ for every state-subformula Ψ of Φ'. Note that the subformula $\neg\mathsf{red} \vee \ldots$ of Φ' should be seen as a state-formula.

When computing a set $Sat(\mathsf{A}\,\varphi)$ write down the corresponding LTL-formula φ' that is checked. However, it is not necessary to perform the LTL-model checking explicitly.

- $Sat(\mathsf{red}) = \{1, 2\}$
- $Sat(\neg\,\mathsf{red}) = \{3, 4, 5, 6\}$
- $Sat(\mathsf{orange}) = \{2, 4, 5\}$
- $Sat(\mathsf{A}\,\neg(\mathsf{orange}\,\mathsf{U}\,\mathsf{X}\,\mathsf{red})) = \{3, 6\}$ (This step involves LTL model checking of the formula $\neg(\mathsf{orange}\,\mathsf{U}\,\mathsf{X}\,\mathsf{red})$.)
- $Sat(\neg\mathsf{A}\,\neg(\mathsf{orange}\,\mathsf{U}\,\mathsf{X}\,\mathsf{red})) = \{1, 2, 4, 5\}$
- $Sat(\neg\mathsf{red} \vee \neg\mathsf{A}\,\neg(\mathsf{orange}\,\mathsf{U}\,\mathsf{X}\,\mathsf{red})) = \{1, 2, 3, 4, 5, 6\}$

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- $Sat(\Phi') = \{1, 2, 3, 4, 5, 6\}$ (This step involves LTL model checking of the formula $\mathsf{G}\, a$ where a is a new atomic proposition representing the state-formula $\neg\mathsf{red} \vee \neg\mathsf{A}\, \neg(\mathsf{orange}\, \mathsf{U}\, \mathsf{X}\, \mathsf{red})$. Hence, all states are labeled with a.)

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Exercise 4 (6 + 6 points)

Consider the LTL formula

$$\varphi = a \,\mathsf{U}\, (\mathsf{X}\,(b \wedge (c \,\mathsf{U}\, \mathsf{X}\, b)) \,\mathsf{U}\, c)$$

The GNBA $\mathcal{A}_\varphi$ is of the form $(\mathcal{Q}, 2^3, q_0, \delta, F_1, F_2, F_3)$.

(i) The set of states $\mathcal{Q}$ is $2^m \cup \{q_0\}$. Determine m by specifying which subformula corresponds to which bit d_i in the state $(d_1, \ldots, d_m)^T$.

$$\begin{pmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \\ d_5 \\ d_6 \\ d_7 \\ d_8 \\ d_9 \end{pmatrix} \sim \begin{pmatrix} a \\ b \\ c \\ \mathsf{X}\, b \\ c \,\mathsf{U}\, \mathsf{X}\, b \\ b \wedge (c \,\mathsf{U}\, \mathsf{X}\, b) \\ \mathsf{X}\,(b \wedge (c \,\mathsf{U}\, \mathsf{X}\, b)) \\ \mathsf{X}\,(b \wedge (c \,\mathsf{U}\, \mathsf{X}\, b)) \,\mathsf{U}\, c \\ \varphi \end{pmatrix}$$

Hence, $m = 9$.

(ii) Suppose F_1 corresponds to the left U of φ, F_2 to the middle U of φ, and F_3 corresponds to the right U of φ. Complete the definitions of F_1, F_2, and F_3.

$$F_1 = \{(d_1, \ldots, d_m)^T \mid d_9 = 0 \vee d_8 = 1\}$$

$$F_2 = \{(d_1, \ldots, d_m)^T \mid d_5 = 0 \vee d_4 = 1\}$$

$$F_3 = \{(d_1, \ldots, d_m)^T \mid d_8 = 0 \vee d_3 = 1\}$$