1. Consider the following (incorrect) definition of the satisfaction relation $\mathcal{M} \models \varphi$ for $\mathcal{V}$-sentences φ that do not contain $=$. (The definition makes use of introduced notation.)
   - $\mathcal{M} \models P(t_1, \dots, t_n)$ iff $(t_1^{\mathcal{M}}, \dots, t_n^{\mathcal{M}}) \in P^{\mathcal{M}}$
   - $\mathcal{M} \models \neg\varphi$ iff $\mathcal{M} \not\models \varphi$.
   - $\mathcal{M} \models \varphi \wedge \psi$ iff $\mathcal{M} \models \varphi$ and $\mathcal{M} \models \psi$.
   - $\mathcal{M} \models \exists x \varphi(x)$ iff $\mathcal{M} \models \varphi(x)$ for some variable x.

   a) Explain briefly why this definition is incorrect. (3 pts)
   b) Extend the given definition so that it becomes a correct one (first-order logic without equality). (5 pts)
   c) Now consider equality and give a formal definition of $\mathcal{M} \models s = t$, where s, t are terms. (3 pts)

2. Consider the following sentences in prenex normal form:
   - $\varphi_1 :\Leftrightarrow \forall x \exists y \forall z \forall u \exists w (Q(x,y,z) \rightarrow P(w,x,y,u))$.
   - $\varphi_2 :\Leftrightarrow \exists x \forall y \forall z \exists w (R(x,z) \wedge R(x,y) \rightarrow (R(x,w) \wedge R(y,w) \wedge R(z,w)))$.
   - $\varphi_3 :\Leftrightarrow \forall x \forall y \exists z \exists u \exists v (S(y,z) \wedge (S(z,u) \wedge (S(x,v) \wedge S(v,u))))$.

   a) Define the Skolemisations φ_i^S of the sentences φ_i, $i = 1,2,3$ given above. (6 pts)
   b) Give the crucial idea of the following claim: *For any SNF formula φ (containing $=$) there exists a formula φ' (without $=$) such that φ is satisfiable iff φ' is satisfiable.* (5 pts)
   c) Give an example of a finite Herbrand structure. (2 pts)

3. In proving the completeness of first-order logic, we started with a consistent theory Γ and defined a complete theory T^+ with $\Gamma \subseteq T^+$. The theory T^+ was defined in stages T_m, $m \geqslant 0$, such that $T_0 = \Gamma$.

   a) Define (in pseudo-code) an algorithm P that based on T_m defines T_{m+1}. (5 pts)
   b) The theory T^+ is satisfiable, explain (briefly) why. (5 pts)

4. Consider the following functions:
   - The addition function $\mathsf{a}(x,y) = x + y$.
   - The (modified) subtraction function $\mathsf{sub}(x,y) = x \dot{-} y$.
   - The factorial function $x!$.

   a) Show that all these functions are recursive by giving explicit definitions. (6 pts)

5. Determine whether the statements on the answer sheet are true or false. Every correct answer is worth 1 points (and every wrong 0 points). (10 pts)

   - Given an embedding $f\colon \mathcal{M} \to \mathcal{N}$, let a be an element in $\mathcal{M}$ and let $\varphi(x)$ be a quantifier-free formula, containing $=$ with one free variable x. Then $\mathcal{N} \models \varphi(f(a))$ doesn't imply $\mathcal{M} \models \varphi(a)$.
   - Every embedding is injective.
   - For any vocabulary $\mathcal{V}$ and any $\mathcal{V}$-structure $\mathcal{M}$, $\mathsf{Th}(\mathcal{M})$ is complete and consistent.
   - For finite $\mathcal{M}$ we have that $\mathcal{M} \cong \mathcal{N}$ if and only if $\mathcal{M} \equiv \mathcal{N}$.
   - For any sentence φ we have: φ is satisfiable if and only if φ has a Herbrand model.
   - Let Γ be a countable set of formulas, if Γ is consistent, then Γ has a countable model.
   - The set of all 17-ary recursive functions is not recursive enumerable.
   - For each n there exists arithmetic sets A that are Σ_n but not Π_n and vice versa.
   - We have $\mathsf{LOGSPACE} \subseteq \mathsf{NLOGSPACE} \subseteq \mathsf{NP} \subseteq \mathsf{PSPACE} \subseteq \mathsf{NPSPACE}$.
   - For any existential second-order sentence φ and any $\mathcal{V}$-structure $\mathcal{M}$ such that $\mathcal{V}$ is finite, the problem $\mathcal{M} \models \varphi$ is decidable by a nondeterministic TM in polynomial time.