1. Consider propositional logic and answer the following questions.
   a) Define the compactness property of propositional logic and decide whether this property is true. (3 pts)
   b) Substantiate your claim that propositional logic is compact, or not compact, by giving a proof sketch. (4 pts)
   c) What is Craig's interpolation theorem and does it hold? Explain your answer briefly. (4 pts)

2. Consider the following (incorrect) definitions of literal embedding and elementary embedding. (The definition makes use of introduced notation.)
   - Let $\mathcal{M}$, $\mathcal{N}$ be $\mathcal{V}$-structures and $f\colon \mathcal{M} \to \mathcal{N}$ a function such that $\mathcal{M} \models \varphi(\vec{a})$ implies $\mathcal{N} \models \varphi(f(\vec{a}))$ where φ is an atomic formula. Then f is called literal embedding.
   - Let $\mathcal{M}$, $\mathcal{N}$ be as above, but suppose in addition that $\mathcal{N} \models \varphi(f(\vec{a}))$ implies $\mathcal{M} \models \varphi(\vec{a})$. Then f is called elementary embedding.

   a) Explain why these definitions are incorrect. (2 pts)
   b) Give the correct definition of literal and elementary embeddings. (4 pts)
   c) Give an extended definition of literal embedding that is still correct. (3 pts)

3. Consider the following clauses $\mathcal{C}$:

$$b = d \qquad a \neq d \qquad (a = b) \vee (a = d) .$$

   a) Decide whether the clause set $\mathcal{C}$ is unsatisfiable. If yes, provide a proof of the empty clause, using the superposition calculus. If no, provide a model. In both cases be as precise as possible, in particular indicate which rules you apply, if you think the formula is unsatisfiable. (4 pts)
   b) Describe Herbrand's method to verify unsatisfiablity of first-order formulas. (5 pts)

4. Let $\mathcal{V}$ be the vocabulary $\{c, f, P\}$, where c is a constant, f is a unary function symbol, and P is a unary relation symbol. Let φ denote the following formula:

$$\forall x(P(x) \to P(f(x))) \wedge P(c) \wedge \exists x \neg P(x) .$$

   a) Give the definition of a Herbrand model for a set of formulas Γ. (3 pts)
   b) Show that φ doesn't have a Herbrand model. (3 pts)
   c) Give a formula ψ, such that ψ is satisfiable if and only if φ is satisfiable. Moreover give a Herbrand model of ψ. (3 pts)

5. Consider the exponentiation function $\exp(x, y) = x^y$, where we set $\exp(0, 0) = 1$. Show that exp is primitive recursive by giving an explicit definition. (2 pts)

6. Determine whether the statements on the answer sheet are true or false. Every correct answer is worth 1 points (and every wrong -1 points). (10 pts)

   – Literal embeddings preserve universal sentences.
   – Any complete theory is consistent.
   – First-order logic is complete, i.e., for any sentence φ, if $\vdash \varphi$, then $\models \varphi$.
   – Let Γ be a countable set of sentences. Then $\Gamma \models \varphi$ if and only if $\Gamma \vdash \varphi$.
   – Let A be a definable subset of $\mathbb{N}$. If A is definable by an existential formula, then A is recursive.
   – The Ackermann function is recursive.
   – For any existential second-order sentence φ and any $\mathcal{V}$-structure $\mathcal{M}$ such that $\mathcal{V}$ is finite, then the problem $\mathcal{M} \models \varphi$ is in PSPACE.
   – The satisfiability problem (of first-order logic) is complete for NP.
   – For finite graphs, reachability is expressible as existential, second-order formula.
   – Second order logic is complete, but not compact.