1. a) *Solution.* See page 45 in the book for the definition; propositional logic is compact. $\square$

   b) *Solution.* See Theorem 1.79 for the proof. $\square$

   c) *Solution.* See Exercise 1.34 for a definition. Propositional logic has the interpolation property. This can be proven directly by induction on the logical complexity of the implication. $\square$

2. a) *Solution.* First, the definition is not well-defined, as the tuple $\vec{a}$ is not specified. Moreover, literal embeddings are defined for literals. With respect to elementary embeddings the definition extends to all formulas $\square$

   b) *Solution.* See Definition 2.50 for the correct definition. $\square$

   c) *Solution.* The following definition would be a correct extension: Let $\mathcal{M}$, $\mathcal{N}$ be $\mathcal{V}$-structures and $f\colon \mathcal{M} \to \mathcal{N}$ a function such that $\mathcal{M} \models \varphi(\vec{a})$ if and only if $\mathcal{N} \models \varphi(f(\vec{a}))$ for any tuple $\vec{a}$ of elements in M. where φ is a quantifier-free formula. Then f is called literal embedding. $\square$

3. a) *Solution.*

$$\dfrac{a \neq d \qquad \dfrac{\dfrac{b = d \quad (a = d) \vee (a = b)}{(a = d) \vee (a = d)}\ \text{superposition right}}{a = d}\ \text{factoring}}{\square}\ \text{resolution}$$

$\square$

   b) *Solution.* See Section 3.3.3 on page 118 in the book. $\square$

4. a) *Solution.* See Definition 3.24 in the book. $\square$

   b) *Solution.* The Herbrand vocabulary $\mathcal{V}_\varphi$ for φ is $\{c, f, P\} = \mathcal{V}$. Hence the Herbrand universe $H(\varphi)$ equals $\{c, f(c), f(f(c)), \dots\}$. By definition any Herbrand structure interprets the constants and functions in $H(\varphi)$ by itself. To satisfy the formula $\forall x(P(x) \to P(f(x)))$ property P has to hold for all elements of $H(\varphi)$. Hence $\exists x \neg P(x)$ cannot be satisfied. $\square$

   c) *Solution.* Use the Skolemisation of φ for the formula ψ. $\square$

5. *Solution.* See Proposition 7.11 in the book $\square$

6.

statement	**yes**	**no**
Literal embeddings preserve universal sentences.	☐	☑
Any complete theory is consistent.	☑	☐
First-order logic is complete, i.e., for any sentence φ, if $\vdash \varphi$, then $\models \varphi$.	☐	☑
Let Γ be a countable set of sentences. Then $\Gamma \models \varphi$ if and only if $\Gamma \vdash \varphi$.	☑	☐
Let A be a definable subset of $\mathbb{N}$. If A is definable by an existential formula, then A is recursive.	☐	☑
The Ackermann function is recursive.	☑	☐
For any existential second-order sentence φ and any $\mathcal{V}$-structure $\mathcal{M}$ such that $\mathcal{V}$ is finite, then the problem $\mathcal{M} \models \varphi$ is in PSPACE.	☑	☐
The satisfiability problem (of first-order logic) is complete for NP.	☐	☑
For finite graphs, reachability is expressible as existential, second-order formula.	☑	☐
Second order logic is complete, but not compact.	☐	☑