Logic (master program)

Georg Moser

Institute of Computer Science @ UIBK

Winter 2008

Schedule

Time and Place

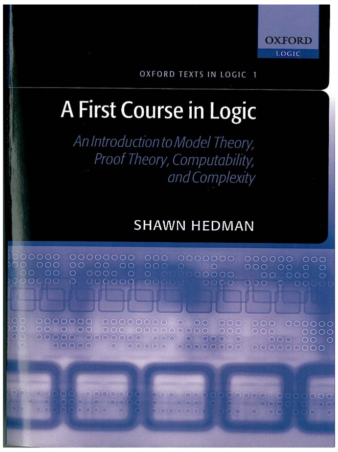
- Thursday, 8:15–9:45, HS E

week 1	October 2	week 8	November 20	
week 2	October 9	week 9	November 27	(LPAR'08)
week 3	October 16	week 10	December 4	
week 4	October 23	week 11	December 11	(FSTTCS'08)
week 5	October 30	week 12	January 8	
week 6	November 6	week 13	January 15	
week 7	November 13	first exam	January 22	

Literature & Online Material

Literature

Shawn Hedman
A First Course in Logic
Oxford University Press, 2008
ISBN 978-0-19-852981-1



Online Material

Transparencies and homework will be available from IP starting with 138.232 after the lecture; exercises and solutions will be discussed during the lecture

Exercises & Exam

- officially there are no exercises as this course is labelled VO
- however, without exercises you'll simply fail the exam
- I'll give bi-weekly exercises which will be discussed in the lecture

Any protest?

Content

week 1	introduction, propositional logic, semantics
week 2	propositional logic, formal proofs, resolution
week 3	homework: propositional logic first-order logic, semantics
week 4	first-order logic, structures, theories and models
week 5	first-order logic, formal proofs, Herbrand theory
week 6	homework: first-order logic, resolution
week 7	completeness of first-order logic, properties of first-order logic
week 8	homework: compactness and completeness introduction to computability
week 9	computability continued, introduction to complexity
week 10	complexity continued, finite model theory
week 11	homework: computability and complexity modal logics in a general setting
week 12	modal logics continued, introduction to Isabelle
week 13	Isabelle frenzy

Content

introduction, propositional logic, semantics, formal proofs, resolution (propositional)

first-order logic, semantics, structures, theories and models, formal proofs, Herbrand theory, resolution (first-order), completeness of first-order logic, properties of first-order logic

introduction to computability, introduction to complexity, finite model theory

beyond first order: modal logics in a general setting, higher-order logics, introduction to Isabelle

There is not only one logic

instead there are many logics: temporal, modal, intuitionistic, fuzzy, dynamic, etc.

Question

why is LICS not enough?

Observation ①

in LICS the following logics have been studied

- propositional logic
- first-order logic
- second-order logic
- temporal logics (LTL, CTL, CTL*)

Logic is a language (not only a tool)

What is a logic?

- a logic is a language equipped with rules for deducing the truth of one sentence from that of another
- a logic is a formal language
- each logic is based on some smallest part usually called atoms or atomic formulas

Example

let n be the smallest natural number that cannot be defined in fewer than 20 words

What is logic?

logic is defined as the study of the principle of reasoning

Logic and Computer Science

Observation ②

- (propositional) logic can be used to effective solve MineSweeper automatically
- logic and computer science form a symbiosis
- the (time) complexity class NP (for graphs) = those decision problems expressible in existential second-order logic

Example

MineSweeper solver & generator
by Christoph Rungg

Example

problem: consider $\{-26, -16, -12, -8, -4, -2, 7, 8, 27\}$, does some subset add up to 10?

the general problem is NP-complete

Content

introduction, propositional logic, semantics, formal proofs, resolution (propositional)

first-order logic, semantics, structures, theories and models, formal proofs, Herbrand theory, resolution (first-order), completeness of first-order logic, properties of first-order logic

introduction to computability, introduction to complexity, finite model theory

beyond first order: modal logics in a general setting, higher-order logics, introduction to Isabelle

Propositional Logic: Syntax

What is propositional logic?

in propositional logic, atomic formulas are propositions

$A =$ "Aristotle is dead" $\quad B =$ "these elections are a catastrophe" ...

Primitive operators

$$\neg \quad > \quad \wedge$$

Definition

formula formation

- the formula $\neg F$ is the negation of F
- the formula $F \wedge G$ is the conjunction of F and G

Demo

Booltool

by Caroline Terzer

Definition

subformula

- any formula is a subformula of itself
- any subformula of F is a subformula of $\neg F$
- any subformula of F and G is a subformula of $(F \wedge G)$

Convention

the formula F and (F) are treated interchangingly

Definition

truth tables

A	$\neg A$
0	1
1	0

A	B	$(A \wedge B)$
0	0	0
0	1	0
1	0	0
1	1	1

Validity, satisfiability, and contradiction

Definition
$\vee$, $\rightarrow$, $\leftrightarrow$

for formulas F, G

- the disjunction $F \vee G$ is defined as $\neg(\neg F \wedge \neg G)$
- the implication $F \rightarrow G$ is defined as $\neg F \vee G$
- the bi-implication $F \leftrightarrow G$ is defined as $((F \rightarrow G) \wedge (G \rightarrow F))$

Definition
assignment

- let $\mathcal{S} = \{A_1, \ldots, A_n\}$ be the set of atomic formulas
- let $\mathcal{F}(\mathcal{S})$ be the set of all formulas over $\mathcal{S}$
- an assignment of $\mathcal{S}$ is a function $\mathcal{A}\colon \mathcal{S} \rightarrow \{0, 1\}$
- an assignment $\mathcal{A}$ extends to an evaluation $\mathcal{A}(F)$ of a formula F

Example
let $\mathcal{A}(A) = 1$, $\mathcal{A}(B) = 0$, then

$$\mathcal{A}(A \wedge B) = 0 \qquad \mathcal{A}(A \wedge (C \vee \neg C)) = 1$$
$$\mathcal{A}(A \wedge (C \vee \neg C))) = 1 \qquad \mathcal{A}(B \vee (C \wedge \neg C)) = 0$$

Definition
models

- if $\mathcal{A}(F) = 1$ then $\mathcal{A}$ models F
- we write $\mathcal{A} \models F$

Definition
valid

- a formula is valid if it holds under every assignment
- if F is valid, we write $\models F$, F is a tautology

Consequence and Equivalence

Definition (satisfiable)

- a formula is satisfiable if $\exists$ assignment that satisfies it
- otherwise, a formula is unsatisfiable
- an unsatisfiable formula is a contradiction

Definition (consequence)

- G is consequence of F, if $\forall$ assignments $\mathcal{A}$, when $\mathcal{A} \models F$, then $\mathcal{A} \models G$
- we write $F \models G$

Lemma

G is consequence of F if and only if $F \to G$ is a tautology

Proof

on black board

Definition (equivalent)

- F is a consequence of G and
- G is a consequence of F, then
- F and G are equivalent
- notation: $F \equiv G$

Example

$$(F \wedge (G \vee H)) \equiv ((F \wedge G) \vee (F \wedge H)) \qquad \neg(F \wedge G) \equiv (\neg F \vee \neg G)$$
$$(F \vee (G \wedge H)) \equiv ((F \vee G) \wedge (F \vee H)) \qquad \neg(F \vee G) \equiv (\neg F \wedge \neg G)$$

Definition

let $\mathcal{F} = \{F_1, F_2, \dots\}$ be a set of formulas

- $\mathcal{A}$ models $\mathcal{F}$ ($\mathcal{A} \models \mathcal{F}$), if $\forall$ F: $\mathcal{A} \models F$
- G is a consequence of $\mathcal{F}$ ($\mathcal{F} \models G$), if $\forall$ $\mathcal{A}$: $\mathcal{A} \models \mathcal{F}$ implies $\mathcal{A} \models G$

Derivability

Example

let $\mathcal{F} = \{A, (A \to B), (B \to C), (C \to D), (D \to E), (E \to F), (F \to G)\}$ such that all formulas in $\mathcal{F}$ are true; hence G is true

Question

how many entries has the truth table for $\bigwedge \mathcal{F} \to G$?

Modus Ponens

$$\frac{X \quad X \to Y}{Y}$$

Example

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{A \quad A \to B}{B} \quad B \to C}{C} \quad C \to D}{D} \quad D \to F}{E} \quad E \to F}{F} \quad F \to G}{G}$$