## Name:

Matr.Nr.:

This test consists of four exercises. Explain your answers. The available points for each item are written in the margin.
[6] 1. Given the functions

```
let rec foldl f b xs = match xs with
    | [] -> b
    | x::xs -> foldl f (f b x) xs
let rec range m n = if m > n then [] else m :: range (m+1) n
```

evaluate the function call foldl (fun ys $x \rightarrow x:: y s$ ) [] (range 1 2) and give at least 6 intermediate steps.
[4] 2. (a) Implement a function remdups : 'a list -> 'a list that removes duplicate elements from a list. E.g.,

$$
\text { remdups }[1 ; 2 ; 1 ; 3]=[2 ; 1 ; 3]
$$

Hint: The function List.mem : 'a -> 'a list -> bool may be useful.
(b) Implement a function pair : 'a list $->$ ('a * 'a) list with the following behavior:

$$
\begin{aligned}
\operatorname{pair}\left[x_{1} ; x_{2} ; x_{3} ; x_{4} ; \ldots ; x_{n}\right] & =\left[\left(x_{1}, x_{2}\right) ;\left(x_{3}, x_{4}\right) ; \ldots ;\left(x_{n-1}, x_{n}\right)\right] \\
\text { pair }\left[x_{1} ; x_{2} ; x_{3}\right] & =\left[\left(x_{1}, x_{2}\right)\right]
\end{aligned}
$$

3. Give the sets $\mathcal{B} \mathcal{V}$ ar, $\mathcal{F} \mathcal{V}$ ar, $\mathcal{V}$ ar, and $\mathcal{S}$ ub for the $\lambda$-term $t=(\lambda a b z . x$ a $(y z))(x y)$.
[6] 4. Rewrite the following $\lambda$-term to NF, giving all intermediate $\beta$-steps.

$$
(\lambda m n f x . m f(n f x))(\lambda f x . f x)(\lambda f x . x)
$$

