University of Innsbruck | **Institute of Computer Science**
Test 1B | **November 27, 2009**

Functional Programming | **WS 2009/2010** | **LVA 703018**

Solutions

This test consists of four exercises. *Explain your answers.* The available points for each item are written in the margin.

[6] **1.** Given the functions

```
let rec foldl f b xs = match xs with
  | []    -> b
  | x::xs -> foldl f (f b x) xs

let rec range m n = if m > n then [] else m :: range (m+1) n
```

evaluate the function call `foldl (fun ys x -> x :: ys) [] (range 1 2)` and give at least 6 intermediate steps.

Solution.

```
foldl (fun ys x -> x :: ys) [] (range 1 2)
 →+ foldl (fun ys x -> x :: ys) [] (1 :: (range 2 2))
 →+ foldl (fun ys x -> x :: ys) [] (1 :: 2 :: (range 3 2))
 →+ foldl (fun ys x -> x :: ys) [] [1; 2]
 →+ foldl (fun ys x -> x :: ys) [1] [2]
 →+ foldl (fun ys x -> x :: ys) [2;1] []
 →+ [2;1]
```

[4] **2.** (a) Implement a function `remdups : 'a list -> 'a list` that removes duplicate elements from a list. E.g.,

`remdups [1;2;1;3]` = `[2;1;3]`

Hint: The function `List.mem : 'a -> 'a list -> bool` may be useful.

Solution.

```
let rec remdups = function
  | []    -> []
  | x::xs -> if List.mem x xs then remdups xs else x :: remdups xs
```

[4] (b) Implement a function `pair : 'a list -> ('a * 'a)list` with the following behavior:

$$\texttt{pair}\ [x_1;x_2;x_3;x_4;\ldots;x_n] = [(x_1,x_2);(x_3,x_4);\ldots;(x_{n-1},x_n)]$$
$$\texttt{pair}\ [x_1;x_2;x_3] = [(x_1,x_2)]$$

Solution.

```
let rec pair = function
  | []       -> []
  | [_]      -> []
  | x::y::xs -> (x,y)::pair xs
```

Solutions

[5] **3.** Give the sets $\mathcal{BV}\mathsf{ar}$, $\mathcal{FV}\mathsf{ar}$, $\mathcal{V}\mathsf{ar}$, and $\mathcal{S}\mathsf{ub}$ for the λ-term $t = (\lambda abz.x\ a\ (y\ z))\ (x\ y)$.

Solution.

$$\begin{aligned}\mathcal{BV}\mathsf{ar} &= \{a, b, z\}\\ \mathcal{FV}\mathsf{ar} &= \{x, y\}\\ \mathcal{V}\mathsf{ar} &= \{a, b, x, y, z\}\\ \mathcal{S}\mathsf{ub} &= \{t, \lambda abz.x\ a\ (y\ z), \lambda bz.x\ a\ (y\ z), \lambda z.x\ a\ (y\ z), x\ a\ (y\ z), x\ a, y\ z, x\ y, a, x, y, z\}\end{aligned}$$

[6] **4.** Rewrite the following λ-term to NF, giving all intermediate β-steps.

$$(\lambda mnfx.m\ f\ (n\ f\ x))\ (\lambda fx.f\ x)\ (\lambda fx.x)$$

Solution.

$$\begin{aligned}&\underline{(\lambda m\ n\ f\ x.m\ f\ (n\ f\ x))\ (\lambda f\ x.f\ x)}\ (\lambda f\ x.x)\\ \rightarrow_\beta\ &\underline{(\lambda n\ f\ x.(\lambda f\ x.f\ x)\ f\ (n\ f\ x))\ (\lambda f\ x.x)}\\ \rightarrow_\beta\ &\lambda f\ x.\underline{(\lambda f\ x.f\ x)\ f}\ ((\lambda f\ x.x)\ f\ x)\\ \rightarrow_\beta\ &\lambda f\ x.\underline{(\lambda x.f\ x)\ ((\lambda f\ x.x)\ f\ x)}\\ \rightarrow_\beta\ &\lambda f\ x.f\ (\underline{(\lambda f\ x.x)\ f}\ x)\\ \rightarrow_\beta\ &\lambda f\ x.f\ (\underline{(\lambda x.x)\ x})\\ \rightarrow_\beta\ &\lambda f\ x.f\ x\end{aligned}$$