First Exam
Game Theory, LVA 703716

March 15, 2010

Name: **Student number**:

The exam consists of 5 exercises with a total of 50 points.

1	2	3	4	5	Sum
☐	☐	☐	☐	☐	☐

1. a) Show that transitivity (of $\succcurlyeq_S$) implies (i) if $f \sim_S g$, $g \sim_S h$, then $g \sim_S h$, and
      (ii) if $f \succ_S g$, $g \succcurlyeq_S h$, then $g \succ_S h$.   (5 pts)

   b) Consider $u\colon X \times \Omega \to \mathbb{R}$, $y \in X$, and the set S of all probabilities $p \in \Delta(\Omega)$
      such that y is optimal. Prove or disprove: The set S is convex.   (6 pts)

2. Consider the following game. Two hunter hunt together for dear or rabbits. If they
   hunt down a dear they get 4 points, if they hunt down a rabbit they get 2 points.
   While they can hunt rabbits on their own, they can only hunt a dear if they work
   together.

   a) Express the game in extensive form.   (5 pts)

   b) Transform the game to fully reduced strategic form.   (5 pts)

3. Consider the following two games:

	x_2	y_2	z_2
x_1	0,4	5,6	8,7
y_1	2,9	6,5	5,1

	x_2	y_2	z_2
x_1	0,0	5,4	4,5
y_1	4,5	0,0	5,4
z_1	5,4	4,5	0,0

   a) Compute all Nash equilibria of the game Γ_1 to the left.   (6 pts)

   b) Compute all Nash equilibria of the game Γ_2 to the right.   (7 pts)

   For the computation use the support enumeration algorithm and note that the
   number of equilibria is odd for both games.

4. Sketch the vertex enumeration algorithm and describe all necessary assumptions
   for its invocation.

5. Determine whether the statements on the answer sheet are true or false. Every correct answer is worth 1 points (and every wrong -1 points). (10 pts)

statement	yes	no
To assert a player is rational, means the player makes decisions consistently in pursuit of her own objective.	☐	☐
A lottery is a function from states to the probability distribution over a set of prizes. If the lottery is independent on the states it depends only on subjective unknowns.	☐	☐
A set of vectors S is convex if for any two vectors p, q also $\lambda p + (1 - \lambda)q \in S$, where $\lambda \in [0, 1]$.	☐	☐
Given a finite game Γ in strategic form, there exists at least one pure equilibrium.	☐	☐
An auction where the bidders have the same private information is called common value auction.	☐	☐
Nash's theorem of the existence of an equilibrium is not extensible to games over infinite strategy sets	☐	☐
A game may have multiple equilibria, but at least one of the equlibria is efficient.	☐	☐
Let $m, n \in \mathbb{N}$ and $m < n$. A two-person game is called degenerated if there exists a strategy profile σ with support size m such that σ has n pure best responses.	☐	☐
For a Nash equilibrium (σ, ρ) of a degenerated two-person game, σ and ρ have support of equal size.	☐	☐
If we can show that $\mathsf{P} = \mathsf{NP}$, then $\mathsf{P} = \mathsf{PPAD}$ follows.	☐	☐