1. a) *Solution.* Suppose $\succcurlyeq_S$ is transitive, that is, for lotteries f, g, and h, if $f \succcurlyeq_S g$, $g \succcurlyeq_S h$, then $f \succcurlyeq_S h$. Below we drop the subscript S to simplify notation.

   Suppose $f \sim g$, $g \sim h$. Then by definition $f \succcurlyeq g$, $g \succcurlyeq h$ and thus by assumption $f \succcurlyeq h$. On the other hand, we have $h \succcurlyeq g$ and $g \succcurlyeq f$, from which $h \succcurlyeq f$ follows. Thus the proof of $f \sim h$ is complete.

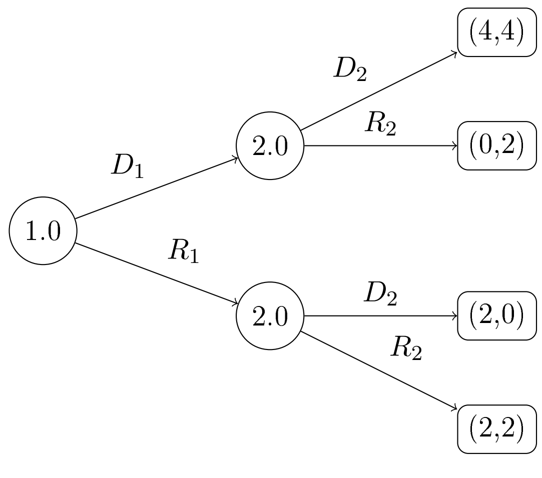
   The proof that $f \succ_S g$, $g \succcurlyeq_S h$, implies $g \succ_S h$ is similar. □

   b) *Solution.*
   – Suppose y is optimal for decision-maker with beliefs p and q
   – $\lambda \in [0,1]$, $r = \lambda p + (1-\lambda)q$

$$\begin{aligned}\sum_{t\in\Omega} r(t)\cdot u(y,t) &\geqslant \lambda \sum_{t\in\Omega} p(t)u(y,t) + (1-\lambda)\sum_{t\in\Omega} q(t)u(y,t) \\ &\geqslant \lambda \sum_{t\in\Omega} p(t)u(x,t) + (1-\lambda)\sum_{t\in\Omega} q(t)u(x,t) \\ &= \sum_{t\in\Omega} r(t)\cdot u(x,t)\end{aligned}$$

□

2. a) *Solution.* We write D_i, R_i $(i = 1,2)$ for the strategies of player i. In extensive form Γ^e, the game is described as follows.



   Importantly the information state is equal for the nodes of player 2, as player 2 cannot observe whether player 1 decided to hunt rabbits, or dears. □

   b) *Solution.* Transforming Γ^e in its normal representation yields the following game Γ:

C_1	C_2	
	D_2	R_2
D_1	4, 4	0, 2
R_1	0, 2	2, 2

It is easy to see that this is also the fully reduced (normal) representation. □

3. a) *Solution.* Consider the game Γ_1 to the left. It is not difficult to see that both games are non-degenerated, that is, no mixed strategy of support size k has more than k best responses. Hence if (x, y) is a Nash equilibrium, then the support of the mixed strategies x, y is equal.

   By considering all possible set of supports, we find the following equilibria:
   - $([x_1], [z_2])$,
   - $([y_1], [x_2])$, and
   - $(\frac{4}{5}[x_1] + \frac{1}{5}[x_2], \frac{3}{4}[y_2] + \frac{1}{4}[z_2])$.

   The argumentation for the pure equilibria is easy. Thus we concentrate on the third equilibria, whose set of support is $\{x_1, y_1\} \times \{y_2, z_2\}$. We write σ_1, σ_2 for the mixed strategies, and get the following equations:

$$5\sigma_2(y_2) + 8\sigma_2(z_2) = 6\sigma_2(y_2) + 5\sigma_2(z_2)$$
$$\sigma_2(y_2) + \sigma_2(z_2) = 1$$
$$6\sigma_1(x_1) + 5\sigma_1(y_1) = 7\sigma_1(x_1) + 1\sigma_(y_1)$$
$$\sigma_1(x_1) + \sigma_1(y_1) = 1$$

   Solving these equations, together with usual side conditions, yields the indicated equilibrium:

$$(\sigma_1, \sigma_2) = (\frac{4}{5}[x_1] + \frac{1}{5}[x_2], \frac{3}{4}[y_2] + \frac{1}{4}[z_2])$$

□

   b) *Solution.* Consider the game Γ_2 to the left. By considering all possible set of supports, we find the following unique equilibrium:
   - $(\frac{1}{3}[x_1] + \frac{1}{3}[y_1] + \frac{1}{3}[z_1], \frac{1}{3}[x_2] + \frac{1}{3}[y_2] + \frac{1}{3}[z_2])$

   Consider the set of support: $\{x_1, y_1, z_1\} \times \{x_2, y_2, z_2\}$, which yields the following equation:

$$5\sigma_2(y_2) + 4\sigma_2(z_2) = 4\sigma_2(x_2) + 5\sigma_2(z_2) = 5\sigma_2(x_2) + 4\sigma_2(y_2)$$
$$\sigma_2(x_2) + \sigma_2(y_2) + \sigma_2(z_2) = 1$$
$$5\sigma_1(y_1) + 4\sigma_1(z_1) = 4\sigma_1(x_1) + 5\sigma_1(z_1) = 5\sigma_1(x_1) + 4\sigma_1(y_1)$$
$$\sigma_1(x_1) + \sigma_1(y_1) + \sigma_1(z_1) = 1$$

   We obtain the above indicated unique solution. □

4. *Solution.* See slides from week 12. □

5.

Solution.

statement	yes	no
To assert a player is rational, means the player makes decisions consistently in pursuit of her own objective.	✓	
A lottery is a function from states to the probability distribution over a set of prizes. If the lottery is independent on the states it depends only on subjective unknowns.		✓
A set of vectors S is convex if for any two vectors p, q also $\lambda p + (1-\lambda)q \in S$, where $\lambda \in [0,1]$.		✓
Given a finite game Γ in strategic form, there exists at least one pure equilibrium.		✓
An auction where the bidders have the same private information is called common value auction.		✓
Nash's theorem of the existence of an equilibrium is not extensible to games over infinite strategy sets		✓
A game may have multiple equilibria, but at least one of the equlibria is efficient.		✓
Let $m, n \in \mathbb{N}$ and $m < n$. A two-person game is called degenerated if there exists a strategy profile σ with support size m such that σ has n pure best responses.	✓	
For a Nash equilibrium (σ, ρ) of a degenerated two-person game, σ and ρ have support of equal size.		✓
If we can show that $\mathsf{P} = \mathsf{NP}$, then $\mathsf{P} = \mathsf{PPAD}$ follows.	✓	

□