## 1 Problem Set 2 (for October 20)

– Show that Transitivity (of  $\succeq_S$ ) implies

if 
$$f \sim_S g$$
 and  $g \sim_S h$  then  $g \sim_S h$   
if  $f \succ_S g$  and  $g \succcurlyeq_S h$  then  $g \succ_S h$ 

- Consider the proof of the Expected Utility Maximisation Theorem. Prove the following equality used:

$$\frac{1}{n} \sum_{t \in \Omega} \sum_{x \in X} f(x|t) \left( u(x,t) \left[ p(t|S)a_1 + (1-p(t|S))a_0 \right] + (1-u(x,t))a_0 \right) = \\
= \frac{1}{n} \sum_{t \in \Omega} \sum_{x \in X} f(x|t)u(x,t)p(t|S)a_1 + \left( 1 - \frac{1}{n} \left( \sum_{t \in \Omega} \sum_{x \in X} f(x|t)u(x,t)p(t|S) \right) \right) a_0 \tag{*}$$

- Consider the proof of the Expected Utility Maximisation Theorem. Prove the reversed direction, i.e., given a utility function u, a conditional-probability function p fulfilling the assertions of the theorem, show that the thus defined relation  $\succeq_S$  fulfils all axioms. (\*)
- A decision-maker expresses the following preference order:

$$[{\textcircled{C}600}] \succ [{\textcircled{C}400}] \succ .9[{\textcircled{C}600}] + .1[{\textcircled{C}0}] \succ .2[{\textcircled{C}600}] + 0.8[{\textcircled{C}0}] \succ .25[{\textcircled{C}400}] + .75[{\textcircled{C}0}] \succ [{\textcircled{C}0}]$$

Prove or disprove: These preferences are consistent with a state-independent utility of money.  $(\ast)$