## 1 Problem Set 4 (for November 17)

- Consider the following voting mechanism: Three committee members decide (vote) each secretly on an option $\alpha, \beta, \gamma$. The the votes are counted. If any options gets two votes, then this option is the outcome. Otherwise player 1 (the chairperson) decides. The payoffs are as follows: If option $\alpha$ is voted, player 1 gets $€ 8$ and player $3 \in 4$, for option $\beta$ player 1 gets $€ 4$ and player 2 gets $€ 8$, and for option $\gamma$, player 2 gets $\in 4$ and player $3 \in 8$. If a player is not metioned in this list, she gets nothing.
(a) Express the game in extensive form.
(b) Transform the game to fully reduced normal representation
- Recall the simple card game discussed in the lecture. Out of 52 cards in the deck, there are 20 cards that are "ten or higher". Suppose the rules are changes so that player 1 wins the money if he has a card that is "ten or higher". Everything else is as before.
(a) Model this game in extensive form and construct the fully reduced normal representation.
- Consider the following game:


Compute all Nash equilibria of the game.

- Suppose that $\Gamma^{\prime}$ is derived from $\Gamma$ by eliminating pure strategies that are strongly dominated in $\Gamma$. Show that $\sigma$ is an equilibrium of $\Gamma$ iff $\sigma$ is an equilibrium of $\Gamma^{\prime}$.
- For each of the following two-player games, find all equilibria. Hint: Each game here has an odd number of equilibria.

|  |  |  | $y_{2}$ |  | $x_{2}$ | $y_{2}$ |  |  | $x_{2}$ | $y_{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $x_{1}$ | 1 | 1,2 | $x_{1}$ | 3, 7 | 6,6 |  | $x_{1}$ | 7, 3 | 6, 6 |
|  | $y_{1}$ |  | 2,1 | $y_{1}$ | 2, 2 | 7,3 |  | $y_{1}$ | 2, 2 | 3,7 |
|  | $x_{2}$ | $y_{2}$ |  |  | $x_{2}$ | $y_{2}$ | $z_{2}$ |  |  |  |
| $x_{1}$ | 0,4 | 5,6 | 8,7 | $x_{1}$ | 0, 0 | 5,4 | 4, 5 |  |  |  |
| $y_{1}$ | 2,9 |  | 5,1 | $y_{1}$ | 4, 5 | 0, 0 | 5,4 0,0 |  |  |  |

