

1 Problem Set 6 (for January 26)

- Consider the following three player game Γ :

C_1	$C_2 \text{ and } C_3$			
	x_3		y_3	
	x_2	y_2	x_2	y_2
x_1	0, 0, 0	6, 5, 4	4, 6, 5	0, 0, 0
y_1	5, 4, 6	0, 0, 0	0, 0, 0	0, 0, 0

- (a) Extend the definition of bi-matrix games to three-matrix games and transform Γ accordingly
 - (b) Find all equilibria of Γ . (*)
- Consider a two-player game given in matrix form where each player has n strategies. Assume that the payoffs for each player are in the range $[0, 1]$ and are selected independently and uniformly at random. Show that the probability this random game is a pure Nash equilibrium approaches $1 - \frac{1}{e}$ as n goes to infinity.
Hint: Recall that $\lim_{n \rightarrow \infty} (1 - \frac{1}{n})^n = \frac{1}{e}$.
- Recall two-person zero-sum games, together with the min-max theorem we considered for these games. Consider a three-person zero-sum game, i.e., a game in which the rewards of the three players always sums to zero. Show that finding a Nash equilibrium in such games is as least as hard as in general two-person games. (*)
- Prove that the support enumeration algorithm leads to unique solutions of the considered linear equations, iff the (two-player) game is non-degenerated. (*)
- Show that in an equilibrium of a non-degenerated game, all pure best responses are played with positive probability.