Game Theory

Georg Moser

Institute of Computer Science @ UIBK

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Summary of Last Lecture

given a game Γ^e in extensive form

Definition

a mixed-strategy profile of Γ^e is any randomised strategy of the normal representation of Γ^e; set of profiles is

$$\prod_{i \in N} \Delta(C_i)$$

Definition

a behavioural-strategy profile of Γ^e is any randomised strategy of the multiagent representation of Γ^e; set of profiles is

$$\prod_{s \in S^*} \Delta(D_s) = \prod_{i \in N} \prod_{s \in S_i} \Delta(D_s)$$

Equilibria in Behavioural Strategies

Definition

a (Nash) equilibrium of an extensive-form game is any equilibrium σ of the multiagent representation such that the mixed representation of σ is an equilibrium of the normal representation

Theorem

if Γ^e admits perfect recall and τ is an equilibrium of the normal representation of Γ^e, then any behavioural representation of τ is an equilibrium (of the multiagent representation of Γ^e)

Theorem

$\forall$ Γ^e with perfect recall $\exists$ a Nash equilibrium (of Γ^e); the perfect recall assumption is needed

Content

motivation, introduction to decision theory, decision theory

basic model of game theory, dominated strategies, common knowledge, Bayesian games, incomplete information, Nash equilibrium

two-person zero-sum games, Bayesian equilibria, sequential equilibra of extensive-form games, subgame-perfect equilibra

(efficient) computation of Nash equilibria, complexity class PPAD, complexity of Nash equilibria, refinements of equilibrium in strategic form, persistent equilibria, games with communication, sender-receiver games

Example

- consider game Γ

C_1	C_2: x_2	y_2
x_1	(3,3)	(3,2)
y_1	(2,2)	(5,6)
z_1	(0,3)	(6,1)

- Γ is representable as two matrices A, B

$$A = \begin{pmatrix} 3 & 3 \\ 2 & 5 \\ 0 & 6 \end{pmatrix} \qquad B = \begin{pmatrix} 3 & 2 \\ 2 & 6 \\ 3 & 1 \end{pmatrix}$$

Notation

- M denotes the set of m pure strategies of player 1
- N denotes the set of n pure strategies of player 2

$$M = \{1, \dots, m\} \qquad N = \{m+1, \dots, m+n\}$$

Notation

- let c be a pure strategy profile and $\sigma \in \prod_{i \in N} \Delta(C_i)$ a randomised strategy profile
- using linear algebra notation, we write:

$$\sigma_i = \sum_{c_i \in C_i} \sigma(c_i)[c_i]$$

- only the vector $x := (\sigma(c_{i1}), \dots, \sigma(c_{i|C_i|}))$ is important
- we call x a mixed strategy

Definition — best response

let x, y be be mixed strategies, then x is best response to y if and only if

$$x_i > 0 \quad \text{implies} \quad (Ay)_i = u = \max\{(Ay)_k \mid k \in M\} \qquad \forall i \in M$$

Definition — support

the support of a mixed strategy x is the set

$$\prod_{i \in N} \{c_i \in C_i \mid x_i > 0\}$$

Example

in the battle of sexes

1. the support of $([f_1],[f_2])$ is $\{f_1\} \times \{f_2\}$ and the support of $([s_1],[s_2])$ is $\{s_1\} \times \{s_2\}$
2. the support of $(0.75[f_1] + 0.25[s_1], 0.25[f_2] + 0.75[s_2])$ is $\{f_1, s_1\} \times \{f_2, s_2\}$

Definition

a (two-player) game is nondegenerate if no mixed strategy of support size k has more than k pure best responses.

Theorem

for a Nash equilibrium (x, y) of a nondegenerated bimatrix game, x and y have support of equal size

Proof

follows from the definition of nondegeneration and best response condition ∎

Equilibria by Support Enumeration

Algorithm

- INPUT: a nondegenerate bimatrix game
- OUTPUT: all Nash equilibria

Method

1. $\forall\ k \in \{1, \dots, \min\{m, n\}\}$
2. $\forall$ k-sized subsets (I, J) of M, N
3. solve the following equation

$$\sum_{i \in I} x_i b_{ij} = v \quad \text{for } j \in J \qquad\qquad \sum_{j \in J} a_{ij} y_j = u \quad \text{for } i \in I$$

$$\sum_{i \in I} x_i = 1 \qquad\qquad \sum_{j \in J} y_j = 1$$

such that $x \geqslant \mathbf{0}$, $y \geqslant \mathbf{0}$ and the best response condition is fulfilled for x and y