Game Theory

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Homework

Problem

Consider a Bayesian game Γ_1 with incomplete information in which player 1 may be either type α or type β. Where player 2 thinks the probability of type α is .9 and the probability of type β is .1. Player 2 has no private information. The payoffs to the two players are shown in the tables below, where the left table asserts $t_1 = \alpha$ and the right $t_1 = \beta$.

	x_2	y_2
x_1	$2, 2$	$-2, 0$
y_1	$0, -2$	$0, 0$

	x_2	y_2
x_1	$0, 2$	$1, 0$
y_1	$1, -2$	$2, 0$

Show the existence of a Bayesian equilibrium in which player 2 chooses x_2.

Problem

Let Γ_2 be a two-person zero-sum game in strategic form. Show that the set

$$\{\sigma_1 \mid \sigma \text{ is an equilibrium of } \Gamma_2\}$$

is a convex subset of the set of randomised strategies for player 1.

Problem

Consider the following three player game Γ:

	C_2 and C_3			
	x_3		y_3	
C_1	x_2	y_2	x_2	y_2
x_1	$0, 0, 0$	$6, 5, 4$	$4, 6, 5$	$0, 0, 0$
y_1	$5, 4, 6$	$0, 0, 0$	$0, 0, 0$	$0, 0, 0$

Find all equilibria of Γ.

Exam Preparation

Last Year's Exams

Question 1

1. Let $X \subseteq \mathbb{R}$ and X be finite with $x \in X$ a prize that amount to € x. Consider the following definition of $f \succcurlyeq_T g$:

$$\min_{s \in T} \sum_{x \in X} x \cdot f(x|s) \geqslant \min_{s \in T} \sum_{x \in X} x \cdot g(x|s) \ .$$

   1. Give an informal explanation of the relation $f \succcurlyeq_T g$.
   2. Does this definition of $\succcurlyeq_T$ violate any of the axioms on decision theory?
   3. Give an example of a preference (perhaps different from above) such that at least one axiom is violated.

Question 2

Consider the following voting mechanism: Three committee members decide (vote) each secretly on an option α, β, γ. The the votes are counted. If any options gets two votes, then this option is the outcome. Otherwise player 1 (the chairperson) decides. The payoffs are as follows: If option α is voted, player 1 gets € 8 and player 3 € 4, for option β player 1 gets € 4 and player 2 gets € 8, and for option γ, player 2 gets € 4 and player 3 € 8. If a player is not metioned in this list, she gets nothing.

1. Express the game in extensive form.
2. Transform the game to reduced strategic form.
3. Formalise the following assertion for games in extensive form as concrete as possible: Whenever a player moves, she remembers all the information she knew earlier..

Question 3

Consider the following two games:

P_1 \ P_2	C	S
C	$-100, -100$	$1, 0$
S	$0, 1$	$0, 0$

Q_1 \ Q_2	M	F
Rr	$0, 0$	$1, -1$
Rp	$0.5, -0.5$	$0, 0$
Pr	$-0.5, 0.5$	$1, -1$
Pp	$0, 0$	$0, 0$

1. Compute all Nash equilibria of the game Γ_1 to the left.
2. Find all strongly dominated strategies of the game Γ_2 to the right. And define the fully reduced normal representation of Γ_2.
3. Compute all Nash equilibria of Γ_2.

Question 4

1. Define the Lemke-Howson algorithm including all necessary assumptions for its invocation.
2. Define the complexity class PPAD and indicate the connection to the LH algorithm.

will replaced by question about Bayesian Nash equilibrium/auctions, ...

Determine whether the statements on the answer sheet are true or false.

statement	yes	no
To assert a player is intelligent, means the player is as smart as the observer.	☐	☐
A randomised strategy σ is a best response to a strategy τ if at least one strategy in the support set of σ is a best responses to τ.	☐	☐
The fully reduced normal representation is derived from the normal representation by eliminating all strategies thar are (randomly) redundant in the normal representation.	☐	☐
A strategy for player i in the Bayesian game is a function from the types of player i into the set of actions (of player i).	☐	☐

statement	yes	no
Given a finite game Γ in extensive form, there exists at least one pure equilibrium.	☐	☐
Baysian Nash equilibria differs slightly from Nash equlibria, in particular Baysian Nash equilibria need not be best responses.	☐	☐
A polyhedron is a polytope that is bounded.	☐	☐
If NP $=$ P, then also PPAD $=$ P.	☐	☐

Two Last Questions

Question
open or closed exam?

Question
exam next week?