Game Theory

Georg Moser

Institute of Computer Science @ UIBK

Winter 2009

Homework

Problem

Let $X \subseteq \mathbb{R}$ and X be finite with $x \in X$ a prize that amount to € x. Consider the following definition of $f \succcurlyeq_T g$:

$$\min_{s \in T} \sum_{x \in X} x \cdot f(x|s) \geqslant \min_{s \in T} \sum_{x \in X} x \cdot g(x|s) \ .$$

1. Give an informal explanation of the relation $f \succcurlyeq_T g$
2. Does this definition of $\succcurlyeq_T$ violate any of the axioms on decision theory?
3. Give an example of a preference (perhaps different from above) such that at least one axiom is violated

Problem

Consider the following four axioms on preferences of decision makers for lotteries f, g, and h and event S:

(i) $f \succcurlyeq_S g$ or $g \succcurlyeq_S f$

(ii) if $e \succ_S f$ and $g \succcurlyeq_S h$, $\alpha \in (0,1]$ then $\alpha e + (1-\alpha)g \succ_S \alpha f + (1-\alpha)h$

(iii) $f \succcurlyeq_S g$ and $g \succcurlyeq_S h$ implies $f \succcurlyeq_S h$

(iv) if $f \succ_S h$ and $0 \leqslant \beta < \alpha \leqslant 1$, then $\alpha f + (1-\alpha)h \succ_S \beta f + (1-\beta)h$

Prove the following two properties.

1. The axiom (iii) follows from the first two
2. The axiom (iv) follows from the first two

Problem

consider the proof of the Expected Utility Maximisation Theorem; prove the following equality used:

$$\frac{1}{n}\sum_{t\in\Omega}\sum_{x\in X} f(x|t)\big(u(x,t)\,[p(t|S)a_1 + (1-p(t|S))a_0] + (1-u(x,t))a_0\big) =$$

$$\frac{1}{n} = \sum_{t\in\Omega}\sum_{x\in X} f(x|t)u(x,t)p(t|S)a_1 +$$

$$+\left(1 - \frac{1}{n}(\sum_{t\in\Omega}\sum_{x\in X} f(x|t)u(x,t)p(t|S))\right) a_0$$

Problem

consider the proof of the Expected Utility Maximisation Theorem. Prove the reveresed direction, i.e., given a utility function u, a conditional-probability function p fulfilling the assertions of the theorem, show that the thus defined relation $\succcurlyeq_S$ fulfils all axioms

Problem

a decision-maker expresses the following preference order:

$$\begin{aligned}[\text{€}600] \succ [\text{€}400] \succ .9[\text{€}600] + .1[\text{€}0] \\ \succ .2[\text{€}600] + 0.8[\text{€}0] \\ \succ .25[\text{€}400] + .75[\text{€}0] \succ [\text{€}0]\end{aligned}$$

Prove or disprove: These preferences are consistent with a state-independent utility of money