1. a) ① $\forall x\ (\mathsf{Dragon}(x) \land \forall y\ (\mathsf{Child}(y,x) \to \mathsf{Happy}(y)) \to \mathsf{Happy}(x))$.
   ② $\forall x\ (\mathsf{Dragon}(x) \to (\mathsf{Fly}(x) \leftrightarrow (\exists y\ \mathsf{Ancestor}(y,x) \land \mathsf{Fly}(y))))$.
   ③ $\forall x\ (\mathsf{Dragon}(x) \land \exists y (\mathsf{Child}(x,y) \land (\mathsf{color}(y) = \mathsf{red})) \to (\mathsf{color}(x) = \mathsf{green}))$.
   ④ $\forall x\ (\mathsf{Dragon}(x) \land (\mathsf{color}(x) = \mathsf{green})) \to \neg\mathsf{Spitfire}(x)$.
   ⑤ $\exists x\ (\mathsf{Dragon}(x) \land (\mathsf{color}(x) = \mathsf{red}) \land \neg\mathsf{Fly}(x))$.

   b) We define a suitable structure $\mathcal{A} = (A, a)$ where $A = \{green, red, grisu\}$ and the mapping a is defined as follows:
   - $a(\mathsf{green}) = green$, $a(\mathsf{red}) = red$
   - $a(\mathsf{color}) = f\colon \{green, red, grisu\} \mapsto red$, that is all function symbols are mapped to the constant function that always returns red.
   - $a(\mathsf{Dragon}) = a(\mathsf{Happy}) = \{grisu\}$ and $a(\mathsf{Fly}) = a(\mathsf{Child}) = a(\mathsf{Ancestor}) = a(\mathsf{Spitfire}) = \varnothing$.

   Let l be an arbitrary assignment and $\mathcal{M} = (\mathcal{A}, l)$. Then $\mathcal{M} \models F$ holds for each of the five sentences above and thus the formalisation is satisfiable.

2. a) See Defintion 2.2 in the lecture notes, page 12.

   b) Consider a language $\mathcal{L} = \{\mathsf{P}\}$ that consists only of a unary predicate constant. For $\mathcal{L}$, we define two structues $\mathcal{A} = (\mathbb{N}, a_{\mathcal{A}})$ and $\mathcal{B} = (\mathbb{N}, a_{\mathcal{B}})$ over the natural numbers, where the mapping $a_{\mathcal{A}}$ inteprets P as the set of all numbers, but $a_{\mathcal{B}}(\mathsf{P}) = \varnothing$. By the above definition we trivially have $\mathcal{A} \cong_1 \mathcal{B}$, but clearly $\mathcal{A} \models \forall x\ \mathsf{P}(x)$, while $\mathcal{B} \not\models \forall x\ \mathsf{P}(x)$.

3. We obtain the following sentences in SNF:

   a) - $F_1^S :\Longleftrightarrow \forall x \forall y\ (\neg(x < y) \lor x < \mathsf{f}(x,y)) \land (\neg(x < y) \lor \mathsf{f}(x,y) < y)$.
      - $F_2^S :\Longleftrightarrow \forall x \forall y\ (\neg\mathsf{P}(y) \lor \mathsf{P}(x))$.
      - $F_3^S :\Longleftrightarrow \forall y\ ((\mathsf{Q}(\mathsf{a}) \lor \mathsf{S}(\mathsf{b})) \land (\neg\mathsf{P}(y) \lor \neg\mathsf{R}(y,\mathsf{a}) \lor \mathsf{S}(\mathsf{b})))$.

   where a, b, f are newly introduced Skolem constants or Skolem functions, respectively.

   b) The claim is incorrect, see Theorem 5.2 on page 37 in the lecture notes for the correct formulation. To see that the claim is wrong consider $F := \exists x\ \mathsf{P}(x)$ and $G := \mathsf{P}(a)$ for some contant a. Observe that the sentence $\exists x\ \mathsf{P}(x) \to \mathsf{P}(a)$ is not valid.

   c) Set $\mathcal{L}' := \mathcal{L} \cup \{\mathsf{d}\}$, where d denotes a new individual constant.

4. a) The formula is unsatisfiable.

   b) In the *ordered* resolution proofs given below the most general unifier employed is written to the right of each applied inference. First we derive the clause $\mathsf{R}(\mathsf{a}, y)$ as follows.

$$\cfrac{\cfrac{\mathsf{P}(x) \lor \mathsf{Q}(x) \lor \mathsf{R}(x,y) \quad \neg\mathsf{P}(x)}{\mathsf{Q}(x) \lor \mathsf{R}(x,y)} \qquad \neg\mathsf{Q}(\mathsf{a})}{\mathsf{R}(\mathsf{a},y)}\ \sigma = \{x \mapsto \mathsf{a}\}$$

Using this deduction Π, we derive the empty clause.

$$\dfrac{\begin{matrix}\Pi\\ \mathsf{R}(\mathsf{a},y)\end{matrix} \qquad \dfrac{\dfrac{\dfrac{\mathsf{S}(\mathsf{a},y)\lor\neg\mathsf{R}(\mathsf{a},y)\lor\mathsf{S}(x,\mathsf{b})}{\mathsf{S}(\mathsf{a},\mathsf{b})\lor\neg\mathsf{R}(\mathsf{a},\mathsf{b})}\,\sigma_1 \qquad \neg\mathsf{S}(\mathsf{a},\mathsf{b})\lor\neg\mathsf{R}(\mathsf{a},\mathsf{b})}{\neg\mathsf{R}(\mathsf{a},\mathsf{b})\lor\neg\mathsf{R}(\mathsf{a},\mathsf{b})}}{\neg\mathsf{R}(\mathsf{a},\mathsf{b})}}{\square}\,\sigma_2$$

Here $\sigma_1 := \{x \mapsto \mathsf{a}, y \mapsto \mathsf{b}\}$ and $\sigma_2 := \{y \mapsto \mathsf{b}\}$.

5.

statement	**yes**	**no**
Let $\mathcal{I}_1, \mathcal{I}_2$ be interpretations such that the respective universes coincide and suppose $\mathcal{I}_1, \mathcal{I}_2$ coincide on the constants in the closed formula F. Then $\mathcal{I}_1 \models F$ iff $\mathcal{I}_2 \models F$.	✓	
For all formulas F and all sets of formulas $\mathcal{G}$ we have that $\mathcal{G} \models F$ iff $\mathsf{Sat}(\mathcal{G} \cup \{\neg F\})$.		✓
Let A, B be sets such that there exists a bijection m between them. Then if $\mathcal{A}$ is a structure with domain A, there exists a structure $\mathcal{B}$ with domain B such that $\mathcal{A} \cong \mathcal{B}$.	✓	
If there exists a finite subset of a set of formulas $\mathcal{G}$ that has a model, then $\mathcal{G}$ has a model.		✓
If a set of formulas $\mathcal{G}$ has an infinite model, then $\mathcal{G}$ also has a countable infinite model.	✓	
If the sentence $A \to C$ holds, then there exists a sentence B such that $A \to B$ and $B \to C$.	✓	
Second-order logic is neither complete, compact, nor satisfies Löwenheim-Skolem.	✓	
Reachability in directed graphs is expressible as existential, first-order formula.		✓
It is undecidable whether two given terms s, t are unifiable.		✓
For any first-order sentence F there exists a set of clauses $\mathcal{C} = \{C_1, \ldots, C_m\}$ such that $F \approx \forall x_1 \ldots \forall x_n (C_1 \land \cdots \land C_m)$.	✓	