Optimizing mkbTT

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Master Seminar 1
Computational Logic Group

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mkbTT on a shoestring

- mkbTT is completion procedure

$\mathcal{E}$ equations	$+$	$\succ$ reduction ordering	$\xrightarrow{\text{kb}}$	$\mathcal{R}$ rewrite system

$\mathcal{R}$ is confluent, terminating, reduced and $\approx_{\mathcal{E}} = \leftrightarrow^*_{\mathcal{R}}$

mkbTT on a shoestring

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$$\underset{\text{equations}}{\mathcal{E}} \quad \underset{\substack{\text{mkbTT} \\ +\text{termination prover}}}{\longrightarrow} \quad \underset{\text{rewrite system}}{\mathcal{R}}$$

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- mkbTT simulates multiple processes
  $P = \{\epsilon, 0, 1, 00, 01, 011, \ldots\}$

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- mkbTT implements inference system on set of nodes

Definition

node is tuple $\langle s : t, R_0, R_1, E, C_0, C_1 \rangle$ of

- terms s, t
- sets of processes $R_0,\ R_1,\ E,\ C_0,\ C_1$

mkbTT on a shoestring

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Definition

node is tuple $\langle s : t, R_0, R_1, E, C_0, C_1 \rangle$ of

- terms s, t — data
- sets of processes R_0, R_1, E, C_0, C_1 — labels

Outline

- Indexing Techniques
- Selection Strategies
- Critical Pair Criteria
- Isomorphisms
- Conclusion

```
procedure mkbTT(N_o, N_c)
if success then
    return p
else if N_o = ∅ then
    fail
else
    n := choose(N_o)
    N_o := rewrite({n}, N_c) ∪ (N_o \ {n})
    if n ≠ ⟨..., ∅, ∅, ∅, ∅, ∅⟩ then
        n := orient(n)
        if n ≠ ⟨..., ∅, ∅, ..., ..., ...⟩ then
            N_o := rewrite(N_c, {n}) ∪ N_o
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            N_c := N_c ∪ {n}
mkbTT(N_o, N_c)
```

rewrite$_1$
$$\frac{N \cup \{\langle s : t, R_1, R_2, E, C_1, C_2\rangle\}}{N \cup \{\langle s : t, R_1 \setminus R, R_2, E \setminus R, C_1, C_2\rangle\} \cup \{\langle s : u, R_1 \cap R, \varnothing, E \cap R, \varnothing, \varnothing\rangle\}}$$
if $\langle l : r, R, \ldots\rangle \in N$, $t \to_{l\to r} u$, $t \doteq l$

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rewrite$_2$

$$\frac{N \cup \{\langle s : t, R_1, R_2, E, C_1, C_2\rangle\}}{\begin{array}{l} N \;\; \cup \{\langle s : t, R_1 \setminus R, R_2 \setminus R, E \setminus R, C_1, C_2\rangle\} \\ \quad\;\; \cup \{\langle s : u, R_1 \cap R, \varnothing, (R_2 \cup E) \cap R, \varnothing, \varnothing\rangle\}\end{array}}$$

if $\langle l : r, R, \ldots\rangle \in N$, $t \rightarrow_{l \rightarrow r} u$, $t \rhd l$

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deduce $\dfrac{\mathcal{N}}{\mathcal{N} \cup \{\langle s : t, \varnothing, \varnothing, R \cap R', \varnothing, \varnothing \rangle\}}$

if $\langle l : r, R, \ldots \rangle, \langle l' : r', R', \ldots \rangle \in \mathcal{N}$
and $s \leftarrow_{l \to r} u \rightarrow_{l' \to r'} t$

The Term Indexing Problem

Given

- a set of terms L
- a binary relation R on terms
- a term t

identify all $s \in L$ with $s\ R\ t$

The Term Indexing Problem

Given

- a set of terms L — index
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- a term t — query term

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Example

mkbTT requires

- variant retrieval in rewrite_1
- encompassment retrieval in rewrite_2
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Term Indexing Techniques

path indexing, discrimination trees, perfect discrimination trees, adaptive automata, code trees, substitution trees, context trees, ...

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Discrimination Trees

Example

- construct index

(1)	$f(g(a,x),c)$	(2)	$f(g(y,b),x)$
(3)	$f(g(a,b),c)$	(4)	$f(g(y,c),b)$
(5)	$f(x,x)$		

Discrimination Trees

Example

- construct index

(1)	$f(g(a,x),c)$	f.g.a.*.c	(2)	$f(g(y,b),x)$	f.g.*.b.*
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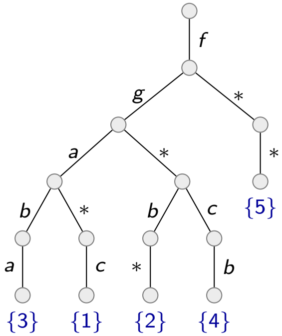

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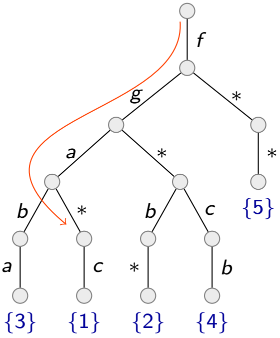
- find terms matching $f(g(a,c),b))$



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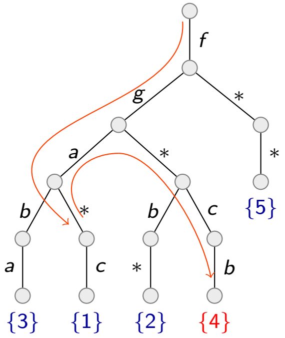

Discrimination Trees

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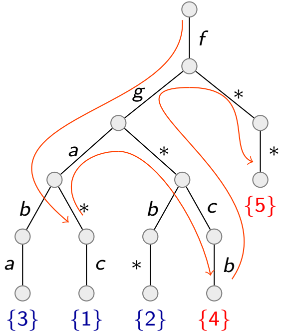
- find terms matching $f(g(a, c), b))$ $f(g(y, c), b), f(x, x)$



Implementation

- path indexing, discrimination trees, code trees for $\rhd$ and $\doteq$
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naive			path indexing			discrimination trees			code trees		
(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)
1990	387	91	1890	345	18	1650	150	5	1580	106	5

(1) total time for successful completions

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(1) total time for successful completions
(2) total time for $\rhd$ retrieval

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Selection Strategies

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  - select process for which $|E_p(\mathcal{N})| + |R_p(\mathcal{N})|$ is minimal
  - select sometimes old, sometimes small node for process

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random

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tuple of node properties is compared lexicographically, minimum chosen

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$p(s_1 : s_2)$ takes s_1 with probability p

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Example (size-age ratio)

`0.9((data(sumsize),?):(*,?))`

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\end{aligned}$$

Example (*sum*)

```
(el(min(e(sum(sizesum))+c(sum(sizesum)))),(data(sizesum),(-el(#),?)))
```

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\textit{node\_property} & ::= & \texttt{*} \mid \texttt{data(}\textit{termpair\_property}\texttt{)} \mid \texttt{el(}\textit{pset\_property}\texttt{)} \\
 & & \mid \texttt{-}\ \textit{node\_property} \mid \textit{node\_property}\ \texttt{+}\ \textit{node\_property} \\
\textit{pset\_property} & ::= & \texttt{\#} \mid \texttt{sum(}\textit{process\_property}\texttt{)} \mid \texttt{min(}\textit{process\_property}\texttt{)} \\
\textit{process\_property} & ::= & \texttt{e(}\textit{eqs\_property}\texttt{)} \mid \texttt{r(}\textit{trs\_property}\texttt{)} \mid \texttt{c(}\textit{trs\_property}\texttt{)} \\
 & & \mid \textit{process\_property}\ \texttt{+}\ \textit{process\_property} \\
\textit{trs\_property} & ::= & \texttt{sum(}\textit{termpair\_property}\texttt{)} \mid \texttt{cp(}\textit{eqs\_property}\texttt{)} \mid \texttt{\#} \\
\textit{eqs\_property} & ::= & \texttt{sum(}\textit{termpair\_property}\texttt{)} \mid \texttt{\#} \\
\textit{termpair\_property} & ::= & \texttt{sizemax} \mid \texttt{sizesum}
\end{array}
$$

Example (*max*)

```
(el(min(e(sum(sizemax))+c(sum(sizemax)))),(data(sizemax),(-el(#),?))))
```

Experiments

	sum		max		slothrop		sa		old	
	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)
CGE_2	138	157	9	43	∞		∞		138	165

Experiments

	sum		max		slothrop		sa		old	
	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)
CGE_2	138	157	9	43	∞		∞		138	165

(1) time in seconds

Experiments

	sum		max		slothrop		sa		old	
	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)
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(1) time in seconds
(2) number of iterations

Experiments

	sum		max		slothrop		sa		old	
	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)
CGE_2	138	157	9	43	∞		∞		138	165
CGE_3	∞		215	56	∞		∞		∞	

(1) time in seconds
(2) number of iterations

Experiments

	sum		max		slothrop		sa		old	
	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)
CGE_2	138	157	9	43	∞		∞		138	165
CGE_3	∞		215	56	∞		∞		∞	
D_8	302	220	∞		∞		∞		∞	

(1) time in seconds
(2) number of iterations

Experiments

	sum		max		slothrop		sa		old	
	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)
CGE_2	138	157	9	43	∞		∞		138	165
CGE_3	∞		215	56	∞		∞		∞	
D_8	302	220	∞		∞		∞		∞	
ASK93-2	∞		∞		∞		281	314	∞	
SK90-3.04	79	133	1.7	38	17	54	∞		57	126

(1) time in seconds
(2) number of iterations

Experiments

	sum		max		slothrop		sa		old	
	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)
CGE_2	138	157	9	43	∞		∞		138	165
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	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)
CGE_2	138	157	9	43	∞		∞		138	165
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SK90-3.04	79	133	1.7	38	17	54	∞		57	126
⋮	⋮		⋮		⋮		⋮		⋮	
# successes	74		71		57		46		77	
time	17863		18753		27806		33485		16217	

(1) time in seconds
(2) number of iterations

Outline

- Indexing Techniques
- Selection Strategies
- Critical Pair Criteria
- Isomorphisms
- Conclusion

```
procedure mkbTT(N_o, N_c)
if success then
    return p
else if N_o = ∅ then
    fail
else
    n := choose(N_o)
    N_o := rewrite({n}, N_c) ∪ (N_o \ {n})
    if n ≠ ⟨..., ∅, ∅, ∅, ∅, ∅⟩ then
        n := orient(n)
        if n ≠ ⟨..., ∅, ∅, ..., ..., ...⟩ then
            N_o := rewrite(N_c, {n}) ∪ N_o
            N_o := deduce(n, N_c) ∪ N_o
            N_c := N_c ∪ {n}
mkbTT(N_o, N_c)
```

deduce $$\frac{\mathcal{N}}{\mathcal{N} \cup \{\langle s : t, \varnothing, \varnothing, R \cap R', \varnothing, \varnothing\rangle\}}$$

if $\langle l : r, R, \ldots\rangle, \langle l' : r', R', \ldots\rangle \in \mathcal{N}$ and $s \leftarrow_{l \to r} u \to_{l' \to r'} t$

```
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mkbTT(N_o, N_c)
```

deduce $\dfrac{\mathcal{N}}{\mathcal{N} \cup \{\langle s : t, \varnothing, \varnothing, R \cap R', \varnothing, \varnothing \rangle\}}$

if $\langle l : r, R, \ldots \rangle, \langle l' : r', R', \ldots \rangle \in \mathcal{N}$
and $s \approx t$ is critical pair

Definition

$\langle l \to r, p, l' \to r' \rangle$ is overlap if

- $p \in \mathrm{Pos}_{\mathcal{F}}(l)$
- mgu σ unifies $l|_p$ and l'
- if $p = \epsilon$ then $l \to r$ and $l' \to r'$ are not variants

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Example

$$\mathcal{R}: \quad \begin{aligned} \sqrt{-x+x} &\to 0 && (1)\\ -0+0 &\to 0 && (2)\\ -0 &\to 0 && (3) \end{aligned}$$

$$CP(\mathcal{R}): \quad \begin{aligned} 0 \leftarrow \sqrt{-0+0} &\to \sqrt{0} && \text{from } \langle (1), 1, (2) \rangle\\ 0 \leftarrow \sqrt{-0+0} &\to \sqrt{0+0} && \text{from } \langle (1), 11, (3) \rangle\\ 0 \leftarrow -0+0 &\to \sqrt{0} && \text{from } \langle (2), 1, (3) \rangle \end{aligned}$$

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Example

$$\mathcal{R}: \quad \begin{array}{rl} \sqrt{-x+x} \to 0 & (1) \\ -0+0 \to 0 & (2) \\ -0 \to 0 & (3) \end{array}$$

all critical pairs required in deduction?

$$CP(\mathcal{R}): \quad \begin{array}{ll} 0 \leftarrow \sqrt{-0+0} \to \sqrt{0} & \text{from } \langle (1), 1, (2) \rangle \\ 0 \leftarrow \sqrt{-0+0} \to \sqrt{0+0} & \text{from } \langle (1), 11, (3) \rangle \\ 0 \leftarrow -0+0 \to \sqrt{0} & \text{from } \langle (2), 1, (3) \rangle \end{array}$$

Critical Pair Criteria in KB

Definition

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redundant CPs

Critical Pair Criteria in KB

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- CPC is mapping such that $\mathsf{CPC}(\mathcal{E}) = \mathcal{E}'$ where $\mathcal{E}' \subseteq \mathcal{E}$
- $$\mathcal{E}_0, \mathcal{R}_0 \vdash_{\mathsf{KB}} \mathcal{E}_1, \mathcal{R}_1 \vdash_{\mathsf{KB}} \mathcal{E}_2, \mathcal{R}_2 \vdash_{\mathsf{KB}} \cdots$$

  is fair with respect to CPC if all equations in $\mathsf{CP}(\mathcal{R}_\infty) \setminus \mathsf{CPC}(\bigcup_i \mathcal{E}_i \cup \mathcal{R}_i)$ are deduced

Critical Pair Criteria in KB

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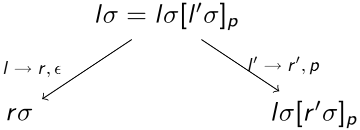
  is fair with respect to CPC if all equations in $\mathrm{CP}(\mathcal{R}_\infty) \setminus \mathrm{CPC}(\bigcup_i \mathcal{E}_i \cup \mathcal{R}_i)$ are deduced
- CPC is correct if for nonfailing derivation $\mathcal{S}$
  $$\mathcal{S} \text{ is fair with respect to CPC} \Longrightarrow \mathcal{S} \text{ is fair}$$

The Primality Criterion

Definition



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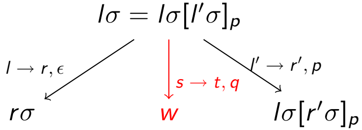

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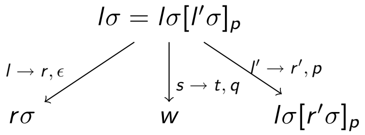

critical pair $r\sigma \approx l\sigma[v\sigma]_p$ is composite if $p < q$

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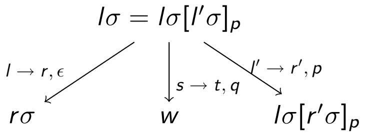

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Example

$$\sqrt{-x + x} \to 0 \qquad (1)$$
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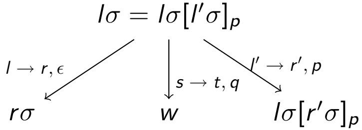

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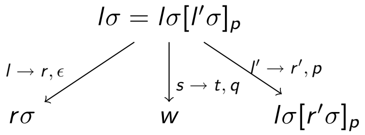

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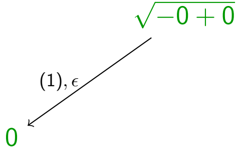

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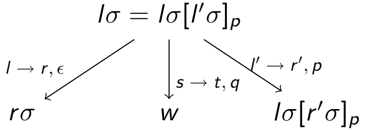

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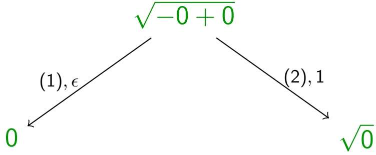

Critical pair $0 \approx \sqrt{0}$

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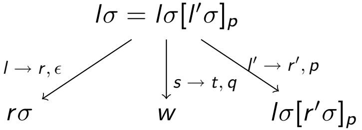

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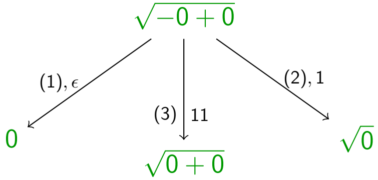

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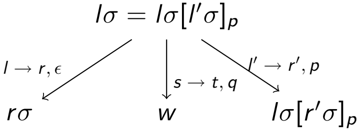

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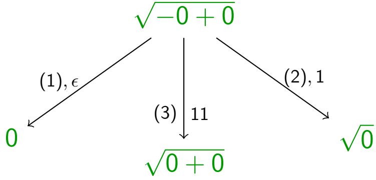

Critical pair $0 \approx \sqrt{0}$ is composite

Critical Pair Criteria in mkbTT

Definition

- if $s \approx t$ is CP then $\mathsf{CPC_m}$ is mapping such that $\mathrm{CPC}_m(s \approx t, P) = P'$ where $P' \subseteq P$

Critical Pair Criteria in mkbTT

Definition

- if $s \approx t$ is CP then CPC$_\mathrm{m}$ is mapping such that $\mathrm{CPC}_m(s \approx t, P) = P'$ where $P' \subseteq P$ processes for which CP is redundant

Critical Pair Criteria in mkbTT

Definition

- if $s \approx t$ is CP then $\mathrm{CPC_m}$ is mapping such that $\mathrm{CPC}_m(s \approx t, P) = P'$ where $P' \subseteq P$
- $$\mathcal{N}_0 \vdash \mathcal{N}_1 \vdash \mathcal{N}_2 \vdash \ldots \vdash \mathcal{N}_k$$

  is fair with respect to CPC_m if projection is fair with respect to CPC for some process $p \in \mathcal{N}_k$

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  is fair with respect to CPC_m if projection is fair with respect to CPC for some process $p \in \mathcal{N}_k$
- obtain $\mathrm{CPC_m}$ from CPC by setting $\mathrm{CPC}_m(s \approx t, P) = Q$ such that $s \approx t \in \mathrm{CPC}(E_p(\mathcal{N}))$ for all $p \in Q$

Critical Pair Criteria in mkbTT

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- if $s \approx t$ is CP then $\mathrm{CPC_m}$ is mapping such that $\mathrm{CPC}_m(s \approx t, P) = P'$ where $P' \subseteq P$
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Lemma

if CPC is correct then also CPC_m is correct

Implementation

- primality criterion PCP — Kapur et al '88
- blocking criterion BCP — Bachmair/Dershowitz '88
- connectedness criterion CCP — Küchlin '85

Implementation

exploit sharing

- primality criterion PCP — Kapur et al '88
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Implementation

- primality criterion PCP — Kapur et al '88
- blocking criterion BCP — Bachmair/Dershowitz '88
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Experiments

none		PCP			BCP			CCP			all		
(1)	(2)	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)
70	189	71	188	416	71	188	391	70	189	155	71	188	468

(1) number of successful completions
(2) average time
(3) number of redundant critical pairs for successful process

Outline

- Indexing Techniques
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mkbTT(N_o, N_c)
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```

Example (Renaming)

- mkbTT run on CGE_2

$\mathcal{N}_0$

$$E_\epsilon = \begin{cases} (x * y) * z \approx x * (y * z) & i(x) * x \approx 1 \\ x * 1 \approx 1 & g(x) * f(y) \approx f(y) * g(x) \\ f(x * y) \approx f(x) * f(y) & g(x * y) \approx g(x) * g(y) \end{cases} \qquad R_\epsilon = C_\epsilon = \varnothing$$

Example (Renaming)

- mkbTT run on CGE_2

$\mathcal{N}_0 \vdash \mathcal{N}_1$

Example (Renaming)

- mkbTT run on CGE_2

$$\mathcal{N}_0 \vdash \mathcal{N}_1 \vdash \mathcal{N}_2$$

Example (Renaming)

- mkbTT run on CGE_2

$$\mathcal{N}_0 \vdash \mathcal{N}_1 \vdash \mathcal{N}_2 \vdash \quad \cdots \quad \vdash \mathcal{N}_i$$

$$E_p = \begin{cases} (x * y) * z \approx x * (y * z) \\ f(1) \approx 1 \\ g(1) \approx 1 \\ g(x) * f(y) \approx f(y) * g(x) \end{cases} \qquad R_p = C_p = \begin{cases} 1 * x \to x \\ i(x) * x \to 1 \\ f(x * y) \to f(x) * f(y) \\ g(x * y) \to g(x) * g(y) \end{cases}$$

Example (Renaming)

- mkbTT run on CGE_2

$$\mathcal{N}_0 \vdash \mathcal{N}_1 \vdash \mathcal{N}_2 \vdash \quad \cdots \quad \vdash \mathcal{N}_i \vdash$$

$$E_p = \begin{cases} (x * y) * z \approx x * (y * z) \\ f(1) \approx 1 \\ g(1) \approx 1 \\ g(x) * f(y) \approx f(y) * g(x) \end{cases} \qquad R_p = C_p = \begin{cases} 1 * x \to x \\ i(x) * x \to 1 \\ f(x * y) \to f(x) * f(y) \\ g(x * y) \to g(x) * g(y) \end{cases}$$

$$\text{orient } \frac{\langle f(x) * g(y) : g(y) * f(x), \varnothing, \varnothing, \{p\}, \varnothing, \varnothing \rangle}{\langle f(x) * g(y) : g(y) * f(x), \{p0\}, \{p1\}, \varnothing, \varnothing, \varnothing \rangle}$$

Example (Renaming)

- mkbTT run on CGE_2

$$\mathcal{N}_0 \vdash \mathcal{N}_1 \vdash \mathcal{N}_2 \vdash \quad \cdots \quad \vdash \mathcal{N}_i \vdash \mathcal{N}_{i+1}$$

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Example (Renaming)

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for rewrite systems $\mathcal{R}, \mathcal{R}'$, mapping $\theta\colon \mathcal{T}(\mathcal{F}, \mathcal{V}) \to \mathcal{T}(\mathcal{F}, \mathcal{V})$ induces isomorphism

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processes p, q are isomorphic in $\mathcal{N}$ if for some θ

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Lemma

if p, q are isomorphic in $\mathcal{N}$ then
$\exists \mathcal{N}' \;\; \mathcal{N} \vdash^* \mathcal{N}'$ with $E_p(\mathcal{N}') = \varnothing \quad \Longleftrightarrow \quad \exists \mathcal{N}'' \;\; \mathcal{N} \vdash^* \mathcal{N}''$ and $E_q(\mathcal{N}'') = \varnothing$

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- $p0, p1$ are isomorphic using $\theta(t) = \begin{cases} f(t') & \text{if } t = g(t') \\ g(t') & \text{if } t = f(t') \\ t & \textit{otherwise} \end{cases}$

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- join $p0, p1$

Example (Argument Permutations)

$\mathcal{N}_0$

$$E_\epsilon = \begin{cases} f(f(x)) \approx x \\ f(x+y) \approx f(x) + f(y) \\ x + (y+z) \approx (x+y)+z \end{cases} \qquad R_\epsilon = C_\epsilon = \varnothing$$

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$\mathcal{N}_0 \vdash \mathcal{N}_1$

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$$\mathcal{N}_0 \vdash \mathcal{N}_1 \vdash \mathcal{N}_2 \vdash \quad \cdots \quad \vdash \mathcal{N}_i$$

$$E_p = \{x + (y + z) \approx (x + y) + z \qquad R_p = C_p = \begin{cases} f(f(x)) \to x \\ f(x + y) \to f(x) + f(y) \end{cases}$$

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$$\mathcal{N}_0 \vdash \mathcal{N}_1 \vdash \mathcal{N}_2 \vdash \quad \cdots \quad \vdash \mathcal{N}_i \vdash$$

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$$\text{orient}\ \frac{\langle x + (y + z) : (x + y) + z, \varnothing, \varnothing, \{p\}, \varnothing, \varnothing \rangle}{\langle x + (y + z) : (x + y) + z, \{p0\}, \{p1\}, \varnothing, \varnothing, \varnothing \rangle}$$

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- isomorphism checks for renamings or argument permutations

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none			renamings			permutations		
(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)
18774	10.9	25.2	18634	8.9	21.1	18841	11.8	25.0

(1) total time
(2) average time for successful completion
(3) average number of processes

Conclusion

Usage

various options can be controlled via web interface

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Conclusion

- indexing techniques pay off
- selection strategies have great impact – optimal one?
- critical pair criteria allow for small improvements
- renaming isomorphisms are very useful for special systems