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## Game Theory and Planning (PhD seminar)

Georg Moser \& Radu Prodan

Institute of Computer Science @ UIBK
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## Schedule

| lectures | seminar talks |
| :--- | :--- |
| October 6(GM) | January 14 |
| October 13 (RP) | January 15 |
| October 20(GM) |  |
| November 3(RP) |  |

## Seminar Talk

the seminar talks should present the main results obtained with respect to the language studied

## Seminar Report

a short, but detailed overview of the material covered in the talk has to be handed in (maximum 5 pages); deadline February 20, 2011

## Topics

- Learning, regret minimisation and equilibria
- Computation of market equilibria by convex programming
- Graphical games
- Mechanism design
- Combinatorial auctions
- Routing games
- Load balancing or job allocation schemes
- Price of anarchy and the design of scalable resource allocation mechanisms
- Cascading behaviour in networks: algorithmic and economic issues
- Sponsored search auctions


## discussed on October 13

## Overview

(Algorithmic) game theory is conceivable as the study of mathematical models of conflict and cooperation between intelligent and rational decision-makers.
Today many applications of computer science involve autonomous decision-makers with conflicting objectives.
One application domain were computer science profi ts from game theoretic knowledge is networks in general and scheduling in particular.

## Games in Extensive Form

## Example

- player 1 and 2 put $1 €$ in a pot
- player 1 draws a card, which is either red or black
- player 1 looks at this card in private and can either raise or pass
- if player 1 passes, then she shows the card
- if the card is red, then player 1 wins the pot
- if the card is black, then player 2 wins the pot
- if player 1 raises, she adds another euro
- player 2 can meet or fold
- if player 2 folds the game ends and player 1 wins the pot
- if player 2 meets she has to add $1 €$
- the games continues as above



## Definition

each decision nodes has two labels

1 the player labelthe information label

Requirement
the set of move-labels following two nodes must be the same if the two nodes are controlled by the same player in the same information state


## Definition

- node 0 is a chance node
- nodes 1,2 are decision nodes
- the path representing the actual events is called path of play


## n-Person Extensive-Form Game

## Definition

an $n$-person extensive-form game $\Gamma^{e}$ is a labelled tree, where also edges are labelled such that

1 each nonterminal node has player label in $\{0,1, \ldots, n\}$ nodes labelled with 0 are called chance nodes nodes labelled within $\{1, \ldots, n\}$ are called decision nodes

2 edges leaving chance nodes (also called alternatives) are labelled with probabilities that sum up to 1

3 player nodes have a second label, the information label reflecting the information stateeach alternative at a player node has a move label

## Definition (cont'd)

6 $\forall$ player $i$,
$\forall$ nodes $x$ y $z$ controlled by $i$,
$\forall$ alternatives $b$ at $x$

- suppose $y$ and $z$ have the same information state $y$ follows $x$ and $b$
- $\exists$ node $w, \exists$ alternative $c$ at $w$ such that $z$ follows $w$ and $c$
- and $w$ is controlled by player $i$ $w$ has the same information label as $x$ $c$ has the same move label as $b$

Question
what does the last condition mean?
Answer
it asserts perfect recall: whenever a player moves, she remembers all the information she knew earlier

## Perfect Information Games

## Definition

if no two nodes have the same information state, we say the game has perfect information

## Definition

- $S_{i}$ is the set of information states per player $i$
- $D_{s}$ is the set of possible moves at $s \in S_{i}$
- the set of strategies for player $i$ is

$$
\prod_{s \in S_{i}} D_{s}=\underbrace{D_{s} \times D_{s} \times \cdot \times D_{s}}_{S_{i-\text { times }}}
$$

## Example

the set of strategies for player $1:\{\operatorname{Rr}, \operatorname{Rp}, \operatorname{Pr}, \operatorname{Pp}\}$

## No Perfect Recall



## Influencing Your Opponent



## Observation

player 1 profits more, if she chooses $T$
player 2 doesn't know player 1's choice


## Observation

player 1 profits more, if she chooses B
player 2 does know player 1's choice

Example


Strategies


## Definition

given a game $\Gamma^{e}$ in extensive form, we define the normal representation as strategic-form game $\Gamma=\left(N,\left(C_{i}\right)_{i \in N},\left(u_{i}\right)_{i \in N}\right)$ :
1 I $N=\{1, \ldots, n\}$, if $\Gamma^{e}$ is an $n$-person game
$\sqrt{2}$ for each $i: C_{i}$ denotes the strategies of each player as defined above
3 we define the expected utility payoff $u_{i}$

- set $C=\prod_{i \in N} C_{i}$
- let $x$ be a node in $\Gamma^{e}$
- let $c \in C$ denote a given strategy profile
- let $P(x \mid c)$ denotes the probability that the path of play goes through $x$, if $c$ is chosen
- let $\Omega^{*}$ denote the set of all terminal nodes
- for $x \in \Omega^{*}, w_{i}(x)$ denotes the payoff for player $i$
- set

$$
u_{i}(c)=\sum_{x \in \Omega^{*}} P(x \mid c) w_{i}(x)
$$

## Strategic-Form Games

## Definition

a strategic-form game is a tuple $\Gamma=\left(N,\left(C_{i}\right)_{i \in N},\left(u_{i}\right)_{i \in N}\right)$ such that
I $N$ is the set of players
2 for each $i: C_{i}$ is the set of strategies of player $i$
3 for each $i: u_{i}: \prod_{i \in N} C_{i} \rightarrow \mathbb{R}$ is the expected utility payoff
a strategic-form game is finite if $N$ and each $C_{i}$ is finite

## Example

consider the card game, suppose player 1 plans to use strategy $R p$ and player 2 plans to use $M$

$$
\begin{aligned}
& u_{1}(R p, M)=2 \cdot \frac{1}{2}+-1 \cdot \frac{1}{2}=0.5 \\
& u_{2}(R p, M)=-2 \cdot \frac{1}{2}+1 \cdot \frac{1}{2}=-0.5
\end{aligned}
$$



## Definition

the normal representation

| $C_{1}$ | $C_{2}$ |  |
| :---: | :---: | :---: |
| $R r$ | 0,0 | $1,-1$ |
| $R p$ | $0.5,-0.5$ | 0,0 |
| $P r$ | $-0.5,0.5$ | $1,-1$ |
| $P p$ | 0,0 | 0,0 |



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## (Fully) Reduced Normal Representation

## Definition

let $\Gamma=\left(N,\left(C_{i}\right)_{i \in N},\left(u_{i}\right)_{i \in N}\right)$, we say $d_{i}$ and $e_{i}$ in $C_{i}$, are payoff equivalent if

$$
u_{j}\left(c_{-i}, d_{i}\right)=u_{j}\left(c_{-i}, e_{i}\right) \quad \text { for all } c_{-i} \in C_{-i}, j \in N
$$

Example
strategies $a_{1} x_{1}, a_{1} y_{1}, a_{1} z_{1}$ are payoff equivalent

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## Definition

identifying payoff equivalent strategies yields the purely reduced normal representation

