University of Innsbruck
1st exam

Institute of Computer Science
January 13, 2009

Introduction to Model Checking (VO) WS 2009/2010 LVA 703503

First name: ______________________________

Last name: ______________________________

Matriculation number: ______________________________

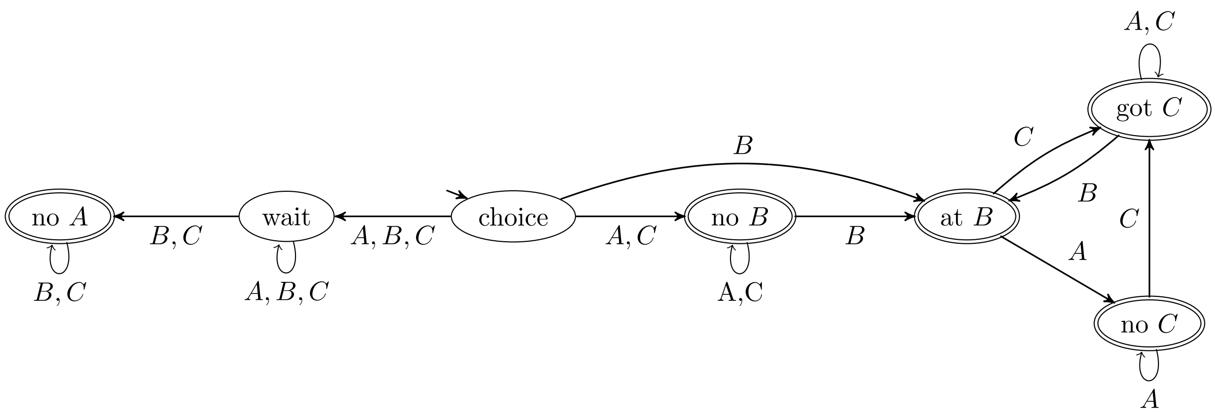
- Please answer all exercises in a readable and precise way.
- Please cross out solution attempts which are replaced by another solution.
- Please do not remove the staples of the exam.
- Cheating is not allowed. Everyone who is caught will fail the exam.

Exercise	Maximal points	Points
1	18	
2	17	
3	26	
4	9	
Σ	70	
Grade		

Exercise 1 (10 + 8 points)

(i) Construct a NBA or GNBA for the following language over $\Sigma = \{A, B, C\}$:

$\{w \in \Sigma^\omega \mid$ if w contains infinitely many A's then between any two successive B's there is at least one $C\}$



(ii) Construct an LTL formula for the following property over $AP = \{\mathsf{a}, \mathsf{b}, \mathsf{c}, \mathsf{d}\}$:

If there are only finitely many a's then there is never a b strictly between a c and a d.

For example, the word $\{\mathsf{a}\}\ \{\mathsf{b}\}\ \{\mathsf{a}\}\ \{\mathsf{c}\}\ \{\mathsf{a}, \mathsf{b}\}\ \{\mathsf{c}\}\ \{\mathsf{a}, \mathsf{d}\}\ \varnothing^\omega$ does not satisfy the property as the second occurrence of the b is strictly between the first c and the d.

$\mathsf{F\,G}\,\neg\mathsf{a} \Rightarrow \mathsf{G}\,\neg(\mathsf{c} \wedge \mathsf{X\,F}\,(\mathsf{b} \wedge \mathsf{X\,F\,d}))$

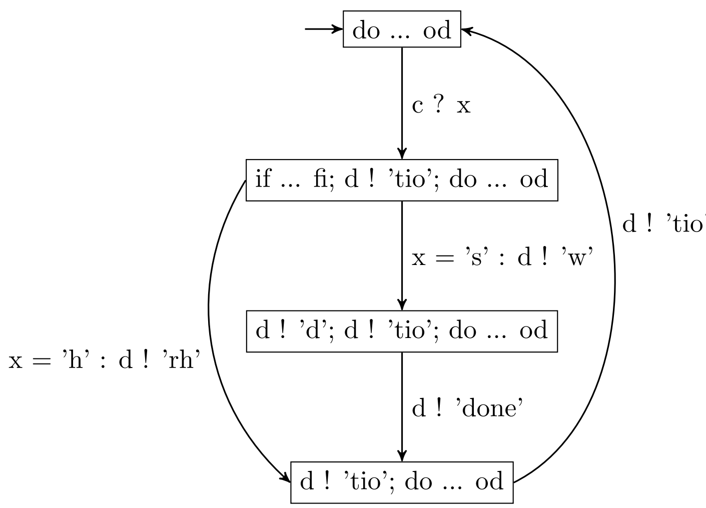
Exercise 2 (17 points)

Consider the following nanoPromela statement.

```
do
  :: true  => c ? x ;
              if :: x = 'help'   => d ! 'raise hand'
                 :: x = 'solved' => d ! 'well'; d ! 'done';
              fi ;
              d ! 'time is over'
od
```

Construct the program graph for this statement. For the first transition this also has to be done formally. You may use abbreviations like "`do` ... `od`", "`if` ... `fi`", "`rh`", "`tio`", ...

$$\dfrac{\dfrac{\dfrac{}{\text{c ? x} \xrightarrow{\text{c ? x}} \text{exit}}}{\text{c ? x; if ... fi; d ! tio} \xrightarrow{\text{c ? x}} \text{if ... fi; d ! tio}}}{\text{do ... od} \xrightarrow{\text{c ? x}} \text{if ... fi; d ! tio; do ... od}}$$



Exercise 3 (6 + 20 points)

Consider the following formula:

$$\varphi = \neg(\text{true}\,\mathsf{U}\,(\neg(\text{true}\,\mathsf{U}\,(\mathsf{req} \wedge \mathsf{X}\,(\text{true}\,\mathsf{U}\,\mathsf{resp})))))$$

The following exercises can be done independently!

(i) Construct a simplified formula ψ with $\varphi \equiv \psi$ by using F and G instead of U. Then try to formulate the meaning of ψ in words (German or English).

$$\varphi \equiv \mathsf{G}\,\mathsf{F}\,(\mathsf{req} \wedge \mathsf{X}\,\mathsf{F}\,\mathsf{resp}) \equiv \mathsf{G}\,\mathsf{F}\,\mathsf{req} \wedge \mathsf{G}\,\mathsf{F}\,\mathsf{resp}$$

There are infinitely many requests and responses.

(ii) Construct the automaton for φ using the improved translation.
$\mathcal{A}_\varphi = (\{q_0\} \uplus 2^6, 2^2, q_0, \delta, F_1, F_2, F_3)$ where

- The reduced Fischer Ladner closure is

$$cl'(\varphi) = \mathsf{req}, \mathsf{resp}, \underbrace{\text{true}\,\mathsf{U}\,\mathsf{resp}}_{\varphi_3}, \underbrace{\mathsf{X}\,\varphi_3}_{\varphi_4}, \underbrace{\text{true}\,\mathsf{U}\,(\mathsf{req} \wedge \varphi_4)}_{\varphi_5}, \text{true}\,\mathsf{U}\,\neg\varphi_5$$

- $(c_1, \ldots, c_6)^T \in \delta(q_0, (d_1, d_2)^T)$ iff $(c_1 \Leftrightarrow d_1) \wedge (c_2 \Leftrightarrow d_2) \wedge \neg c_6$

- $(c_1, \ldots, c_6)^T \in \delta((b_1, \ldots, b_6)^T, (d_1, d_2)^T)$ iff $(c_1 \Leftrightarrow d_1) \wedge (c_2 \Leftrightarrow d_2) \wedge (b_3 \Leftrightarrow b_2 \vee c_3) \wedge (b_4 \Leftrightarrow c_3) \wedge (b_5 \Leftrightarrow (b_1 \wedge b_4) \vee c_5) \wedge (b_6 \Leftrightarrow \neg b_5 \vee c_6)$

- $F_1 = \{(b_1, \ldots, b_6)^T \mid \neg b_3 \vee b_2\}$
  $F_2 = \{(b_1, \ldots, b_6)^T \mid \neg b_5 \vee (b_1 \wedge b_4)\}$
  $F_3 = \{(b_1, \ldots, b_6)^T \mid \neg b_6 \vee \neg b_5\}$

Compute the number of direct successor states of state $(1, 0, 0, 0, 1, 0)^T$. Simplifying the conditions yields $c_3 = c_6 = 0, c_5 = 1$, hence there are 8 states $(*, *, 0, *, 1, 0)$.

Exercise 4 (9 points)

Each correct answer is worth 3 points. A wrong answer results in zero points (for that question, not for the whole exercise). Giving no answer is worth 1 point.

	Yes	No
$\mathsf{F}\,(a \,\mathsf{U}\, b) \wedge \mathsf{G}\,\neg a$ is satisfiable.	✓	
If $\mathcal{A}_1$ has n_1 states and $\mathcal{A}_2$ has n_2 states then $\mathcal{A}_{\mathcal{A}_1 \cap \mathcal{A}_2}$ has $n_1 + n_2$ states.		✓
As LTL model checking is in PSPACE, for every LTL formula φ of size n there is a corresponding NBA for φ which has polynomial size in n.		✓