

**First name:** \_\_\_\_\_

**Last name:** \_\_\_\_\_

**Matriculation number:** \_\_\_\_\_

- Please answer all exercises in a readable and precise way.
- Please cross out solution attempts which are replaced by another solution.
- Please do not remove the staples of the exam.
- Cheating is not allowed. Everyone who is caught will fail the exam.

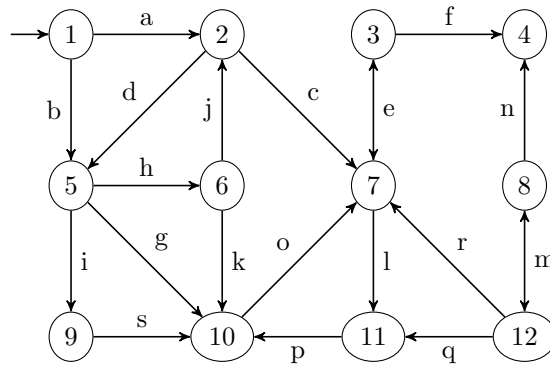
| Exercise | Maximal points | Points |
|----------|----------------|--------|
| 1        | 18             |        |
| 2        | 18             |        |
| 3        | 25             |        |
| 4        | 9              |        |
| $\Sigma$ | 70             |        |
| Grade    |                |        |

### Exercise 1 (9 + 9 points)

(i) Consider the GNBA  $\mathcal{A}$  over  $\Sigma = \{a, \dots, s\}$  where the 4 final state sets are

- $F_1 = \{1, 2, 3, 9, 10, 12\}$
- $F_2 = \{5, 6, 7, 8, 12\}$
- $F_3 = \{1, 4, 11, 12\}$
- $F_4 = \{2, 4, 6, 8, 10, 12\}$

and where the structure of the graph is as follows.



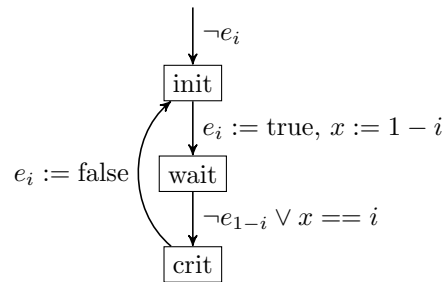
Use the algorithm to check emptiness of GNBA to determine whether  $\mathcal{L}(\mathcal{A}) = \emptyset$ . If  $\mathcal{L}(\mathcal{A}) \neq \emptyset$ , also give an accepted word that the algorithm produces.

(ii) Let  $\Sigma = \{0, 1\}$ . Formalize the following language over  $\Sigma$  as NBA.

$$\mathcal{L} = \{w \in \Sigma^\omega \mid w \neq (1100)^\omega\}$$

## Exercise 2 (18 points)

Consider the channel system  $[process_0 \mid process_1]$  where for  $process_i$  we have the following program graph:



Construct the reachable part for the corresponding transition system where the states are five-tuples of the following form:

(location process 0, location process 1, value  $e_0$ , value  $e_1$ , value  $x$ )

You do neither have to provide the set of atomic propositions nor the labeling function. For specifying states, you may use abbreviations like  $(i, c, f, t, 0)$  for  $(init, crit, false, true, 0)$ , etc.

### Exercise 3 (6 + 19 points)

Consider the following formula:

$$\varphi = \neg(\text{true } \mathbf{U} (\mathbf{a} \wedge \neg \mathbf{X} \mathbf{a})) \wedge \text{true } \mathbf{U} \mathbf{a} \wedge \text{true } \mathbf{U} \neg \mathbf{a}$$

The following exercises can be done independently!

- (i) Construct a simplified formula  $\psi$  with  $\varphi \equiv \psi$  by using  $\mathbf{F}$  and  $\mathbf{G}$  instead of  $\mathbf{U}$ . Then try to formulate the meaning of  $\psi$  in words (German or English).

- (ii) Construct the automaton for  $\varphi$  using the improved translation.

$\mathcal{A}_\varphi = (\{q_0\} \uplus 2^5, 2^1, q_0, \delta, F_1, F_2, F_3)$  where

- The reduced Fischer Ladner closure is

$$cl'(\varphi) =$$

- $(c_1, \dots, c_5)^T \in \delta(q_0, d_1)$  iff

- $(c_1, \dots, c_5)^T \in \delta((b_1, \dots, b_5)^T, d_1)$  iff

- $F_1 = \{(b_1, \dots, b_5)^T \mid$

$$F_2 = \{(b_1, \dots, b_5)^T \mid$$

$$F_3 = \{(b_1, \dots, b_5)^T \mid$$

Explicitly give all outgoing transitions of state  $(1, 1, 1, 1, 1)^T$ .

**Exercise 4 (9 points)**

Each correct answer is worth 3 points. A wrong answer results in zero points (for that question, not for the whole exercise). Giving no answer is worth 1 point.

|  | Yes | No |
|--|-----|----|
| Let $\varphi$ be a formula with $n^2$ temporal operators. Every equivalent GNBA has a size of at least $2^n$ states.   |     |    |
| $X\varphi \cup \varphi \equiv X\varphi \vee \varphi$   |     |    |
| Let <code>loop</code> be an abbreviation for <code>do :: b =&gt; c ! x; c ? y od</code> . The set of sub-statements of <code>loop</code> is <code>{loop, c ? y; loop}</code> . |     |    |