Introduction to Model Checking (VO) — WS 2010/2011 — LVA 703503

First name: ______________________

Last name: ______________________

Matriculation number: ______________________

- Please answer all exercises in a readable and precise way.
- Please cross out solution attempts which are replaced by another solution.
- Please do not remove the staples of the exam.
- Cheating is not allowed. Everyone who is caught will fail the exam.

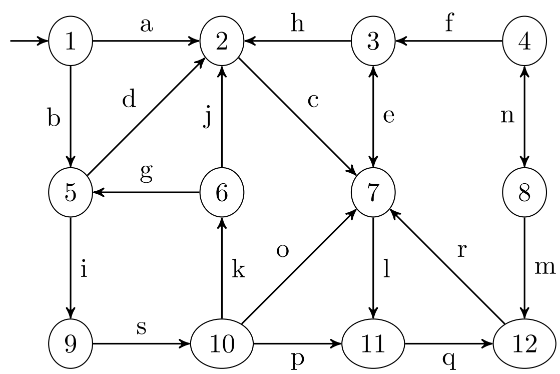
Exercise	Maximal points	Points
1	16	
2	21	
3	24	
4	9	
Σ	70	
Grade		

Exercise 1 (8 + 8 points)

(i) Consider the GNBA $\mathcal{A}$ over $\Sigma = \{a, \ldots, s\}$ where the 3 final state sets are

- $F_1 = \{1, 2, 3, 4, 5, 6\}$
- $F_2 = \{2, 4, 6, 8, 10, 12\}$
- $F_3 = \{3, 4, 7, 8, 11, 12\}$

and where the structure of the graph is as follows.



Use the algorithm to check emptyness of GNBAs to determine whether $\mathcal{L}(\mathcal{A}) = \varnothing$. If $\mathcal{L}(\mathcal{A}) \neq \varnothing$, also give an accepted word that the algorithm produces.

(ii) Let $\Sigma = \{0, 1, 2\}$. Formalize the following language over Σ as NBA.

$$\mathcal{L} = \{w \in \Sigma^\omega \mid \text{if } w \text{ contains infinitely many 0s, then every 1 is followed by a 2 some times later in } w\}$$

Exercise 2 (21 points)

Consider the following nanoPromela statement.

```
do
  :: a=1 => if :: b=2  => c ! 3 fi ;
            d ! 4
  :: e=5 => c ? x
od
```

Construct the program graph for this statement. For the whole statement, additionally derive all transitions formally using the inference rules.

You may use abbreviations like "`do` ... `od`", "`if` ... `fi`", ...

Exercise 3 (3 + 21 points)

Consider the following formula:

$$\varphi = \neg(\text{true} \,\mathsf{U}\, (\mathsf{r} \wedge (\neg(\mathsf{r} \,\mathsf{U}\, (\mathsf{g} \wedge \mathsf{X}\, (\mathsf{g} \,\mathsf{U}\, \mathsf{r}))))))$$

(i) Construct a simplified formula ψ with $\varphi \equiv \psi$ where all negations are eliminated.

(ii) Construct the automaton for φ using the improved translation.
$\mathcal{A}_\varphi = (\{q_0\} \uplus 2^6, 2^2, q_0, \delta, F_1, F_2, F_3)$ where

- The reduced Fischer Ladner closure is

  $cl'(\varphi) = \mathsf{r}, \mathsf{g},$

- $(c_1, \ldots, c_6)^T \in \delta(q_0, (d_1, d_2)^T)$ iff

- $(c_1, \ldots, c_6)^T \in \delta((b_1, \ldots, b_6)^T, (d_1, d_2)^T)$ iff

- $F_1 = \{(b_1, \ldots, b_6)^T \mid$

  $F_2 = \{(b_1, \ldots, b_6)^T \mid$

  $F_3 = \{(b_1, \ldots, b_6)^T \mid$

Explicitly give all incoming transitions of state $(0, 0, 0, 0, 0, 0)^T$.

Exercise 4 (9 points)

Prove the equivalence $\mathsf{X}\,\varphi\,\mathsf{U}\,\varphi \equiv \varphi \vee \mathsf{X}\,\varphi$.