Introduction to Model Checking (VO) | WS 2010/2011 | LVA 703503

First name: ______________________

Last name: ______________________

Matriculation number: ______________________

- Please answer all exercises in a readable and precise way.
- Please cross out solution attempts which are replaced by another solution.
- Please do not remove the staples of the exam.
- Cheating is not allowed. Everyone who is caught will fail the exam.

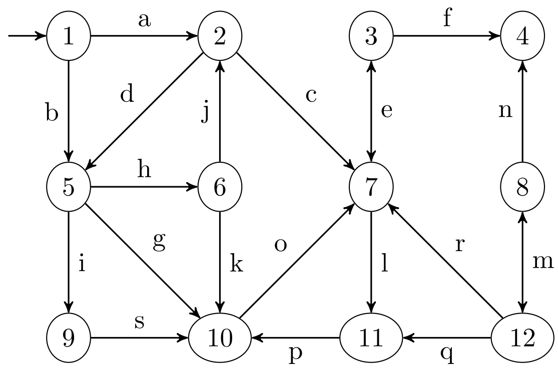
Exercise	Maximal points	Points
1	18	
2	18	
3	25	
4	9	
Σ	70	
Grade		

Exercise 1 (9 + 9 points)

(i) Consider the GNBA $\mathcal{A}$ over $\Sigma = \{a, \ldots, s\}$ where the 4 final state sets are

- $F_1 = \{1, 2, 3, 9, 10, 12\}$
- $F_2 = \{5, 6, 7, 8, 12\}$
- $F_3 = \{1, 4, 11, 12\}$
- $F_4 = \{2, 4, 6, 8, 10, 12\}$

and where the structure of the graph is as follows.

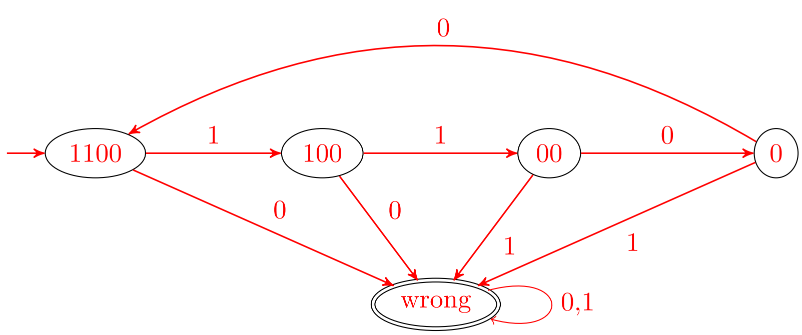


Use the algorithm to check emptyness of GNBAs to determine whether $\mathcal{L}(\mathcal{A}) = \varnothing$. If $\mathcal{L}(\mathcal{A}) \neq \varnothing$, also give an accepted word that the algorithm produces.

The SCCs of the graph are $\{2, 5, 6\}$, $\{3, 7, 10, 11\}$, and $\{8, 12\}$. Although 12 is in every final state set, the SCC $\{8, 12\}$ does not yield an accepting word, as it is not reachable. Also SCC $\{2, 5, 6\}$ does not yield an accepting word, since it does not contain a state of F_3. However, SCC $\{3, 7, 10, 11\}$ is reachable and contains a final state from each F_i. Hence, $\mathcal{L}(\mathcal{A}) \neq \varnothing$ and the word is constructed by constructing a path to some node in $\{3, 7, 10, 11\}$, and then continuing with a path which traverses all nodes in the SCC. Hence, the word $ac(eelpo)^\omega$ might be the possible output of the algorithm.

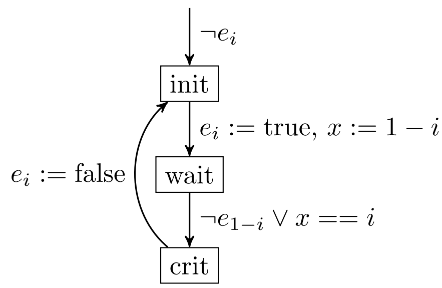
(ii) Let $\Sigma = \{0, 1\}$. Formalize the following language over Σ as NBA.

$$\mathcal{L} = \{w \in \Sigma^\omega \mid w \neq (1100)^\omega\}$$



Exercise 2 (18 points)

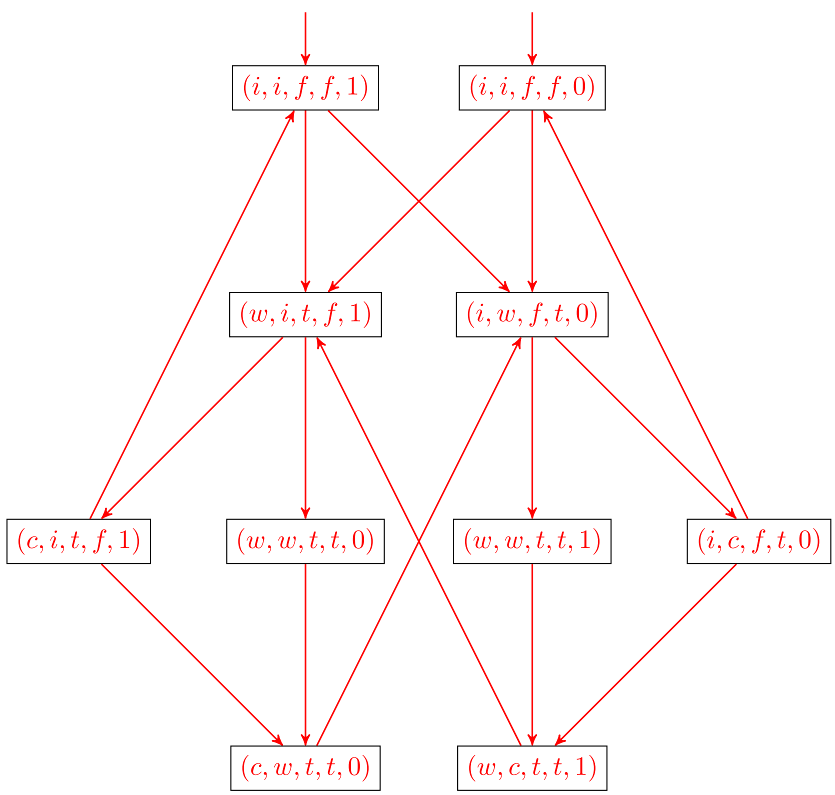
Consider the channel system $[process_0 \mid process_1]$ where for $process_i$ we have the following program graph:



Construct the reachable part for the corresponding transition system where the states are five-tuples of the following form:

$$(\text{location process } 0, \text{ location process } 1, \text{ value } e_0, \text{ value } e_1, \text{ value } x)$$

You do neither have to provide the set of atomic propositions nor the labeling function. For specifying states, you may use abbreviations like $(i, c, f, t, 0)$ for $(init, crit, \text{false}, \text{true}, 0)$, etc.



Exercise 3 (6 + 19 points)

Consider the following formula:

$$\varphi = \neg(\text{true}\,\mathsf{U}\,(\mathsf{a} \land \neg\mathsf{X}\,\mathsf{a})) \land \text{true}\,\mathsf{U}\,\mathsf{a} \land \text{true}\,\mathsf{U}\,\neg\mathsf{a}$$

The following exercises can be done independently!

(i) Construct a simplified formula ψ with $\varphi \equiv \psi$ by using F and G instead of U. Then try to formulate the meaning of ψ in words (German or English).

$$\begin{aligned}
\varphi &\equiv \neg(\mathsf{F}\,(\mathsf{a} \land \neg\mathsf{X}\,\mathsf{a})) \land \mathsf{F}\,\mathsf{a} \land \mathsf{F}\,\neg\mathsf{a} \\
&\equiv \mathsf{G}\,(\neg(\mathsf{a} \land \neg\mathsf{X}\,\mathsf{a})) \land \mathsf{F}\,\mathsf{a} \land \mathsf{F}\,\neg\mathsf{a} \\
&\equiv \mathsf{G}\,(\mathsf{a} \Rightarrow \mathsf{X}\,\mathsf{a}) \land \mathsf{F}\,\mathsf{a} \land \mathsf{F}\,\neg\mathsf{a} \\
&\equiv \mathsf{G}\,(\mathsf{a} \Rightarrow \mathsf{G}\,\mathsf{a}) \land \mathsf{F}\,\mathsf{a} \land \neg\mathsf{a} \\
(&\equiv \neg\mathsf{a} \land \neg\mathsf{a}\,\mathsf{U}\,\mathsf{G}\,\mathsf{a})
\end{aligned}$$

a is not satisfied in the first moment, but later on there will be a moment where a is satisfied. Moreover, after the first occurrence of a, a will always be satisfied.

(ii) Construct the automaton for φ using the improved translation.
$\mathcal{A}_\varphi = (\{q_0\} \uplus 2^5, 2^1, q_0, \delta, F_1, F_2, F_3)$ where

- The reduced Fischer Ladner closure is

$$cl'(\varphi) = \mathsf{a}, \mathsf{X}\,\mathsf{a}, \text{true}\,\mathsf{U}\,(\mathsf{a} \land \neg\mathsf{X}\,\mathsf{a}), \text{true}\,\mathsf{U}\,\mathsf{a}, \text{true}\,\mathsf{U}\,\neg\mathsf{a}$$

- $(c_1, \ldots, c_5)^T \in \delta(q_0, d_1)$ iff $(c_1 \Leftrightarrow d_1) \land \neg c_3 \land c_4 \land c_5$

- $(c_1, \ldots, c_5)^T \in \delta((b_1, \ldots, b_5)^T, d_1)$ iff $(c_1 \Leftrightarrow d_1) \land (b_2 \Leftrightarrow c_1) \land (b_3 \Leftrightarrow ((b_1 \land \neg b_2) \lor c_3)) \land (b_4 \Leftrightarrow (b_1 \lor c_4)) \land (b_5 \Leftrightarrow (\neg b_1 \lor c_5))$

- $F_1 = \{(b_1, \ldots, b_5)^T \mid \neg b_3 \lor (b_1 \land \neg b_2)\}$
  $F_2 = \{(b_1, \ldots, b_5)^T \mid \neg b_4 \lor b_1\}$
  $F_3 = \{(b_1, \ldots, b_5)^T \mid \neg b_5 \lor \neg b_1\}$

Explicitly give all outgoing transitions of state $(1,1,1,1,1)^T$. Simplifying the conditions yields $c_1 = c_3 = c_5 = d_1 = 1$, hence there are 4 transitions $(1,1,1,1,1)^T \xrightarrow{1} (1,*,1,*,1)^T$.

Exercise 4 (9 points)

Each correct answer is worth 3 points. A wrong answer results in zero points (for that question, not for the whole exercise). Giving no answer is worth 1 point.

	Yes	No
Let φ be a formula with n^2 temporal operators. Every equivalent GNBA has a size of at least 2^n states.		✓
$\mathsf{X}\,\varphi\,\mathsf{U}\,\varphi \equiv \mathsf{X}\,\varphi \vee \varphi$	✓	
Let `loop` be an abbreviation for `do :: b => c ! x; c ? y od`. The set of sub-statements of `loop` is {`loop`, `c ? y; loop`}.		✓