Introduction to Model Checking (VO) | WS 2010/2011 | LVA 703503

First name: ____________________

Last name: ____________________

Matriculation number: ____________________

- Please answer all exercises in a readable and precise way.
- Please cross out solution attempts which are replaced by another solution.
- Please do not remove the staples of the exam.
- Cheating is not allowed. Everyone who is caught will fail the exam.

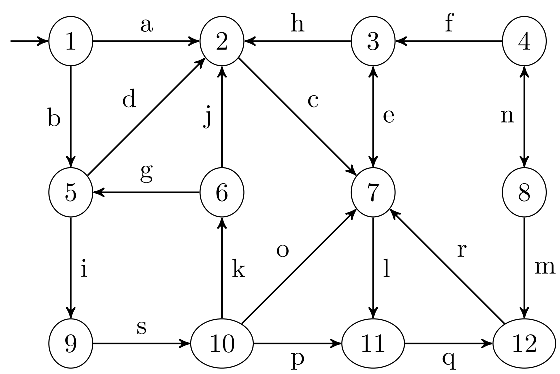
Exercise	Maximal points	Points
1	16	
2	21	
3	24	
4	9	
Σ	70	
Grade		

Exercise 1 (8 + 8 points)

(i) Consider the GNBA $\mathcal{A}$ over $\Sigma = \{a, \dots, s\}$ where the 3 final state sets are

- $F_1 = \{1, 2, 3, 4, 5, 6\}$
- $F_2 = \{2, 4, 6, 8, 10, 12\}$
- $F_3 = \{3, 4, 7, 8, 11, 12\}$

and where the structure of the graph is as follows.

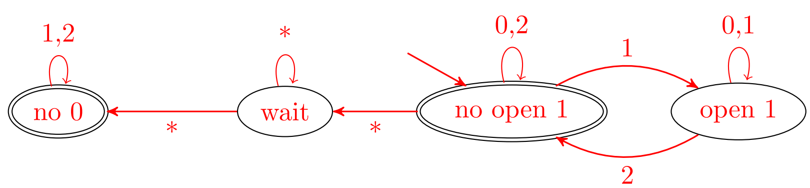


Use the algorithm to check emptyness of GNBAs to determine whether $\mathcal{L}(\mathcal{A}) = \varnothing$. If $\mathcal{L}(\mathcal{A}) \neq \varnothing$, also give an accepted word that the algorithm produces.

The SCCs of the graph are $\{5, 6, 9, 10\}$, $\{2, 3, 7, 11, 12\}$, and $\{4, 8\}$. Only SCC $\{2, 3, 7, 11, 12\}$ is reachable and contains states of all three final state sets. Hence, $\mathcal{L}(\mathcal{A}) \neq \varnothing$ and the word is constructed by constructing a path to some node in $\{2, 3, 7, 11, 12\}$, and then continuing with a path which traverses all nodes in the SCC. Hence, the word $a(clqreh)^\omega$ might be a possible output of the algorithm.

(ii) Let $\Sigma = \{0, 1, 2\}$. Formalize the following language over Σ as NBA.

$$\mathcal{L} = \{w \in \Sigma^\omega \mid \text{if } w \text{ contains infinitely many 0s, then every 1 is followed by a 2 some times later in } w\}$$



Exercise 2 (21 points)

Consider the following nanoPromela statement.

```
do
  :: a=1 => if :: b=2  => c ! 3 fi ;
            d ! 4
  :: e=5 => c ? x
od
```

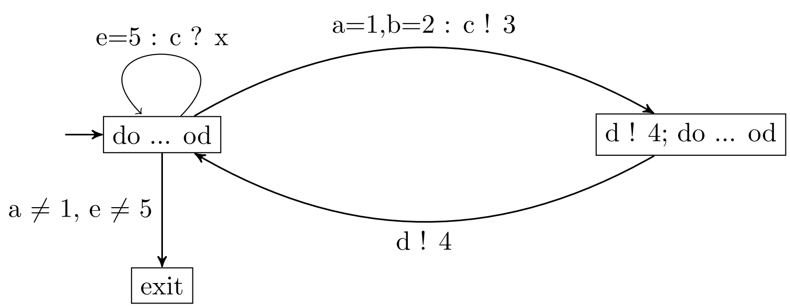
Construct the program graph for this statement. For the whole statement, additionally derive all transitions formally using the inference rules.

You may use abbreviations like "`do ... od`", "`if ... fi`", ...

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\text{c ! 3} \xrightarrow{\text{c ! 3}} \text{exit}}}{\text{if ... fi} \xrightarrow{\text{b = 2 : c ! 3}} \text{exit}}}{\text{if ... fi; d ! 4} \xrightarrow{\text{b = 2 : c ! 3}} \text{d ! 4}}}{\text{do ... od} \xrightarrow{\text{a = 1, b = 2 : c ! 3}} \text{d ! 4; do ... od}}$$

$$\dfrac{\dfrac{}{\text{c ? x} \xrightarrow{\text{c ? x}} \text{exit}}}{\text{do ... od} \xrightarrow{\text{e = 5 : c ? x}} \text{do ... od}}$$

$$\dfrac{}{\text{do ... od} \xrightarrow{\text{a} \neq \text{1, e} \neq \text{5}} \text{exit}}$$



Exercise 3 (3 + 21 points)

Consider the following formula:

$$\varphi = \neg(\mathrm{true}\,\mathsf{U}\,(\mathsf{r} \land (\neg(\mathsf{r}\,\mathsf{U}\,(\mathsf{g} \land \mathsf{X}\,(\mathsf{g}\,\mathsf{U}\,\mathsf{r}))))))$$

(i) Construct a simplified formula ψ with $\varphi \equiv \psi$ where all negations are eliminated.

$$\begin{aligned}\varphi &\equiv \neg(\mathsf{F}\,(\mathsf{r} \land (\neg(\mathsf{r}\,\mathsf{U}\,(\mathsf{g} \land \mathsf{X}\,(\mathsf{g}\,\mathsf{U}\,\mathsf{r}))))))\\ &\equiv \mathsf{G}\,\neg(\mathsf{r} \land (\neg(\mathsf{r}\,\mathsf{U}\,(\mathsf{g} \land \mathsf{X}\,(\mathsf{g}\,\mathsf{U}\,\mathsf{r})))))\\ &\equiv \mathsf{G}\,(\neg\mathsf{r} \lor \mathsf{r}\,\mathsf{U}\,(\mathsf{g} \land \mathsf{X}\,(\mathsf{g}\,\mathsf{U}\,\mathsf{r})))\\ &\equiv \mathsf{G}\,(\mathsf{r} \Rightarrow \mathsf{r}\,\mathsf{U}\,(\mathsf{g} \land \mathsf{X}\,(\mathsf{g}\,\mathsf{U}\,\mathsf{r})))\end{aligned}$$

(ii) Construct the automaton for φ using the improved translation.
$\mathcal{A}_\varphi = (\{q_0\} \uplus 2^6, 2^2, q_0, \delta, F_1, F_2, F_3)$ where

- The reduced Fischer Ladner closure is

$$cl'(\varphi) = \mathsf{r}, \mathsf{g}, \underbrace{\mathsf{g}\,\mathsf{U}\,\mathsf{r}}_{\varphi_3}, \underbrace{\mathsf{X}\,\varphi_3}_{\varphi_4}, \underbrace{\mathsf{r}\,\mathsf{U}\,(\mathsf{g} \land \varphi_4)}_{\varphi_5}, \underbrace{\mathrm{true}\,\mathsf{U}\,(\mathsf{r} \land \neg\varphi_5)}_{\varphi_6}$$

- $(c_1, \dots, c_6)^T \in \delta(q_0, (d_1, d_2)^T)$ iff $(c_1 \Leftrightarrow d_1) \land (c_2 \Leftrightarrow d_2) \land \neg c_6$

- $(c_1, \dots, c_6)^T \in \delta((b_1, \dots, b_6)^T, (d_1, d_2)^T)$ iff $(c_1 \Leftrightarrow d_1) \land (c_2 \Leftrightarrow d_2) \land (b_3 \Leftrightarrow (b_1 \lor (b_2 \land c_3))) \land (b_4 \Leftrightarrow c_3) \land (b_5 \Leftrightarrow ((b_2 \land b_4) \lor (b_1 \land c_5))) \land (b_6 \Leftrightarrow ((b_1 \land \neg b_5) \lor c_6))$

- $F_1 = \{(b_1, \dots, b_6)^T \mid \neg b_3 \lor b_1\}$
  $F_2 = \{(b_1, \dots, b_6)^T \mid \neg b_5 \lor (b_2 \land b_4)\}$
  $F_3 = \{(b_1, \dots, b_6)^T \mid \neg b_6 \lor (b_1 \land \neg b_5)\}$

Explicitly give all incoming transitions of state $(0,0,0,0,0,0)^T$. Simplifying the conditions yields $b_1 = b_3 = b_6, d_1 = d_2 = b_4 = b_5 = 0$, hence there are 4 transitions $(b_1, *, b_1, 0, 0, b_1)^T \xrightarrow{(0,0)} (0,0,0,0,0,0)^T$. Moreover, there is the transition $q_0 \xrightarrow{(0,0)} (0,0,0,0,0,0)$

Exercise 4 (9 points)

Prove the equivalence $\mathsf{X}\,\varphi\,\mathsf{U}\,\varphi \equiv \varphi \vee \mathsf{X}\,\varphi$.

$$
\begin{aligned}
&\mathsf{X}\,\varphi\,\mathsf{U}\,\varphi \\
&\equiv \varphi \vee (\mathsf{X}\,\varphi \wedge \mathsf{X}\,(\mathsf{X}\,\varphi\,\mathsf{U}\,\varphi)) && (\mathsf{U}\text{-expansion}) \\
&\equiv \varphi \vee (\mathsf{X}\,\varphi \wedge \mathsf{X}\,(\varphi \vee (\mathsf{X}\,\varphi \wedge \mathsf{X}\,(\mathsf{X}\,\varphi\,\mathsf{U}\,\varphi)))) && (\mathsf{U}\text{-expansion}) \\
&\equiv \varphi \vee (\mathsf{X}\,\varphi \wedge (\mathsf{X}\,\varphi \vee \mathsf{X}\,(\mathsf{X}\,\varphi \wedge \mathsf{X}\,(\mathsf{X}\,\varphi\,\mathsf{U}\,\varphi)))) && (\mathsf{X}\text{-distribution}) \\
&\equiv \varphi \vee ((\mathsf{X}\,\varphi \wedge \mathsf{X}\,\varphi) \vee (\mathsf{X}\,\varphi \wedge \mathsf{X}\,(\mathsf{X}\,\varphi \wedge \mathsf{X}\,(\mathsf{X}\,\varphi\,\mathsf{U}\,\varphi)))) && (\text{distribution}) \\
&\equiv \varphi \vee (\mathsf{X}\,\varphi \vee (\mathsf{X}\,\varphi \wedge \mathsf{X}\,(\mathsf{X}\,\varphi \wedge \mathsf{X}\,(\mathsf{X}\,\varphi\,\mathsf{U}\,\varphi)))) && (\text{idempotency}) \\
&\equiv \varphi \vee \mathsf{X}\,\varphi && (\mathsf{X}\,\varphi \vee (\mathsf{X}\,\varphi \wedge \ldots) \equiv \mathsf{X}\,\varphi)
\end{aligned}
$$