Introduction to Model Checking (VO) | WS 2010/2011 | LVA 703503

First name: ______________________

Last name: ______________________

Matriculation number: ______________________

- Please answer all exercises in a readable and precise way.
- Please cross out solution attempts which are replaced by another solution.
- Please do not remove the staples of the exam.
- Cheating is not allowed. Everyone who is caught will fail the exam.

Exercise	Maximal points	Points
1	**16**	
2	**21**	
3	**21**	
4	**12**	
Σ	**70**	
Grade		

Exercise 1 (16 points)

We consider an extension of LTL with a yesterday operator Y where $\mathsf{Y}\,\varphi$ should be true iff φ was true one time moment before. Moreover, at the beginning of the timeline, $\mathsf{Y}\,\varphi$ is never true.

In the following example the semantics of Y is illustrated via a φ-expansion. Here, the truth values for a are determined by the input word.

time	0	1	2	3	4	5	6	7	8
a	1	0	0	1	0	0	1	1	
$\mathsf{Y}\,\mathsf{a}$	0	1	0	0	1	0	0	1	1

For each of the following formulas, provide an equivalent LTL formula without Y which accepts the same set of infinite words (without proof).

(i) $\mathsf{F}\,\mathsf{Y}\,\mathsf{a} \equiv \mathsf{F}\,\mathsf{a}$

(ii) $\mathsf{Y}\,\mathsf{F}\,\mathsf{a} \equiv \text{false}$

(iii) $\mathsf{G}\,(\mathsf{Y}\,\mathsf{a} \Rightarrow \mathsf{b}) \equiv \mathsf{G}\,(\mathsf{a} \Rightarrow \mathsf{X}\,\mathsf{b})$

(iv) $\mathsf{G}\,(\mathsf{a} \Rightarrow \mathsf{Y}\,\mathsf{b}) \equiv \mathsf{G}\,(\mathsf{X}\,\mathsf{a} \Rightarrow \mathsf{b}) \wedge \neg\mathsf{a}$

Exercise 2 (21 points)

Consider the following nanoPromela statement.

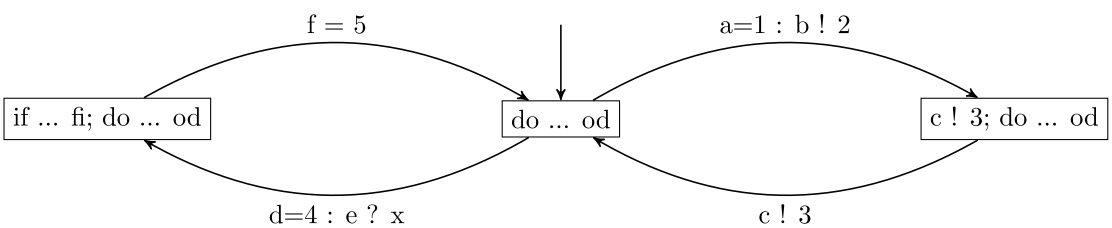
```
do
  :: true => if :: a=1 => b ! 2 fi ;
             c ! 3
  :: d=4 => e ? x;
            if :: f=5 => skip fi
od
```

Construct the program graph for this statement. For the whole statement, additionally derive all transitions formally using the inference rules.

You may use abbreviations like “`do ... od`”, “`if ... fi`”, . . .

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\text{b ! 2} \xrightarrow{\text{b ! 2}} \text{exit}}}{\text{if ... fi} \xrightarrow{\text{a = 1 : b ! 2}} \text{exit}}}{\text{if ... fi; c ! 3} \xrightarrow{\text{a = 1 : b ! 2}} \text{c ! 3}}}{\text{do ... od} \xrightarrow{\text{a = 1 : b ! 2}} \text{c ! 3; do ... od}}$$

$$\dfrac{\dfrac{\dfrac{}{\text{e ? x} \xrightarrow{\text{e ? x}} \text{exit}}}{\text{e ? x; if ... fi} \xrightarrow{\text{e ? x}} \text{if ... fi}}}{\text{do ... od} \xrightarrow{\text{d = 4: e ? x}} \text{if :: f = 5 => skip fi; do ... od}}$$



Exercise 3 (5 + 16 points)

Consider the following formula:

$$\varphi = \mathsf{X}\,(\mathsf{b}\,\mathsf{U}\,\mathsf{a})\,\mathsf{U}\,(\mathsf{X}\,\mathsf{a})$$

(i) Fill in all values of the φ-expansion for the given input word that can uniquely be determined where ... may be arbitrary.

a	0	1	0	0	1	0	0	0	0	...
b	0	1	1	1	0	1	0	1	1	...
b U a	0	1	1	1	1	0	0	–	–	
X (b U a)	1	1	1	1	0	0	–	–	–	
X a	1	0	0	1	0	0	0	0	–	
φ	1	1	1	1	0	0	–	–	–	

(ii) Construct the automaton for φ.
$\mathcal{A}_\varphi = (\{q_0\} \uplus 2^6, 2^2, q_0, \delta, F_1, F_2)$ where the Fischer Ladner closure is $cl(\varphi) = \mathsf{a}, \mathsf{b}, \underbrace{\mathsf{b}\,\mathsf{U}\,\mathsf{a}}_{\varphi_3}, \underbrace{\mathsf{X}\,(\mathsf{b}\,\mathsf{U}\,\mathsf{a})}_{\varphi_4}, \underbrace{\mathsf{X}\,\mathsf{a}}_{\varphi_5}, \varphi$

- $(c_1, \ldots, c_6)^T \in \delta(q_0, (d_1, d_2)^T)$ iff $(c_1 \Leftrightarrow d_1) \wedge (c_2 \Leftrightarrow d_2) \wedge c_6$

- $(c_1, \ldots, c_6)^T \in \delta((b_1, \ldots, b_6)^T, (d_1, d_2)^T)$ iff $(c_1 \Leftrightarrow d_1) \wedge (c_2 \Leftrightarrow d_2) \wedge (b_3 \Leftrightarrow b_1 \vee (b_2 \wedge c_3)) \wedge (b_4 \Leftrightarrow c_3) \wedge (b_5 \Leftrightarrow c_1) \wedge (b_6 \Leftrightarrow b_5 \vee (b_4 \wedge c_6))$

- $F_1 = \{(b_1, \ldots, b_6)^T \mid \neg b_3 \vee b_1\}$
  $F_2 = \{(b_1, \ldots, b_6)^T \mid \neg b_6 \vee b_5\}$

Explicitly list all transitions leading to state $q = (0,0,1,0,0,1)^T$. Simplifying the conditions yields $\neg d_1 \wedge \neg d_2 \wedge (b_3 \Leftrightarrow b_1 \vee b_2) \wedge b_4 \wedge \neg b_5 \wedge b_6$. Hence,

$$\begin{array}{rcl}
(0,0,0,1,0,1)^T & \xrightarrow{(0,0)^T} & q \\
(0,1,1,1,0,1)^T & \xrightarrow{(0,0)^T} & q \\
(1,0,1,1,0,1)^T & \xrightarrow{(0,0)^T} & q \\
(1,1,1,1,0,1)^T & \xrightarrow{(0,0)^T} & q \\
q_0 & \xrightarrow{(0,0)^T} & q
\end{array}$$

Exercise 4 (12 points)

Each correct answer is worth 3 points. A wrong answer results in zero points (for that question, not for the whole exercise). Giving no answer is worth 1 point.

	Yes	No
Let $\mathcal{A}$ be a GNBA with final state sets $F_1, \ldots, F_k$. Let $\mathcal{B}$ be as $\mathcal{A}$ where additionally for every i, j with $1 \leqslant i < j \leqslant k$, $F_i \cup F_j$ is added as new final state set to $\mathcal{B}$. Then $\mathcal{L}(\mathcal{A}) = \mathcal{L}(\mathcal{B})$.	✓	
$\varphi \mathsf{U} \mathsf{X} \varphi \equiv \varphi \vee \mathsf{X} \varphi$		✓
LTL model checking is PSPACE hard.	✓	
In a channel system with a channel of capacity $c > 0$, sending and receiving is done simultaneously.		✓