1. Consider the following propositional formulas

$$A := \neg\left((q \vee r) \wedge (p \to q) \to (r \to \neg q) \vee (p \wedge r)\right)$$
$$B := (p \wedge q \to r) \to (p \to (q \to r))$$

   a) Show that formula A is satisfiable by giving a satisfying assignment. (4 pts)
   b) Show that formula B is valid, by giving a proof in natural deduction. (6 pts)

2. Consider the following sentences:

   ① A bird is large if its father or its mother is large.
   ② A bird can fly if all its relatives can fly.
   ③ Any bird is related to its father and its mother.
   ④ Birds eat fish if they do not eat worms.
   ⑤ There exists a large bird that cannot fly.

   a) For each of the sentences above, give a first-order formula (perhaps with equality) that formalises the sentence. Use therefore the following constants, functions and predicates:
      - Individual constants: fish, worms.
      - Function constants: father, mother, which are unary.
      - Predicates constants: Bird, Small, Medium, Large, Fly, which are unary and Relative, Eat, which are binary.

      The interpretation of the unary predicates follows their names, $\mathsf{Relative}(x, y)$ represents that "x is a relative of y", while $\mathsf{Eat}(x, y)$ means "x eats y". (5 pts)
   b) Show that your formalisation is satisfiable. (3 pts)

3. Consider first-order logic *without* equality. Let $\mathcal{I}$ be an interpretation and F a formula. Then we attempt to define the satisfaction relation $\mathcal{I} \models F$ as follows:
   - $\mathcal{I} \models P(t_1, \ldots, t_n)$ iff $(t_1^{\mathcal{I}}, \ldots, t_n^{\mathcal{I}}) \in P^{\mathcal{I}}$
   - $\mathcal{I} \models \neg F$ iff $\mathcal{I} \not\models F$.
   - $\mathcal{I} \models F \wedge G$ iff $\mathcal{I} \models F$ and $\mathcal{I} \models G$.
   - $\mathcal{I} \models \exists x F(x)$ iff there exists a variable x such that $\mathcal{I} \models F(x)$

   a) The definition is incorrect. Give an example or explain informally why this definition is incorrect. (3 pts)
   b) Strictly speaking this definition is also incomplete for the language of first-order logic without equality, explain why. (2 pts)
   c) Extend the given definition so that it becomes correct. (5 pts)

4. Consider the following first-order formulas with predicate constants P, Q, and R:

$$C := \forall x \exists y \forall z \forall u \exists w\, (\mathsf{Q}(x, y, z) \to \mathsf{P}(w, x, y, u))$$
$$D := \exists x \forall y \forall z \exists w\, (\mathsf{R}(x, z) \wedge \mathsf{R}(x, y) \to \mathsf{R}(x, w) \wedge \mathsf{R}(y, w) \wedge \mathsf{R}(z, w))$$
$$E := \forall x(\neg \mathsf{Q}(x) \to \mathsf{R}(x)) \to \neg(\forall x \neg \mathsf{R}(x) \wedge \exists x \neg \mathsf{Q}(x))$$

a) Give the SNF of formula C. (3 pts)

b) Give the SNF of formula D. (3 pts)

a) Use resolution for first-order to show that formula E is valid. (6 pts)

5. Determine whether the following statements are true or false. Give your answers on the answer sheet. Every correct answer is worth 1 points and *every wrong -1 points.* (10 pts)

   - Consider propositional logic. Then $A_1, \ldots, A_n \models B$, asserts that $\mathsf{v}(B) = \mathsf{T}$, whenever there exists $i \in \{1, \ldots, n\}$ such that $\mathsf{v}(A_i) = \mathsf{T}$, for any assignment v.
   - Natural deduction for propositional logic is sound and complete. Furthermore it is the only formal system with these properties.
   - Let $\mathcal{A}$, $\mathcal{B}$ be first-order structures such that $\mathcal{A} \cong \mathcal{B}$. Then for every sentence F we have $\mathcal{A} \models F$ iff $\mathcal{B} \models F$.
   - If every finite subset of a set of first-order formulas $\mathcal{G}$ has a countable model, then $\mathcal{G}$ has a countable model.
   - Suppose $\mathcal{G}$ is a set of first-order formulas and $\mathcal{G} \vdash F$. Then there exists a finite subset $\mathcal{G}_0 \subseteq \mathcal{G}$ such that $\mathcal{G}_0 \vdash F$.
   - Let S be the set of satisfiable sets of first-order formulas $\mathcal{G}$. Then S fulfils the satisfaction properties.
   - Let $\mathcal{G}$ be a set of first-order formulas and let F be a first-order formula such that $\mathcal{G} \vdash F$. Then $\mathcal{G} \models \neg F$.
   - There exists a satisfiable and universal first-order sentence (without $=$) F, such that F doesn't have a Herbrand model.
   - A unifier σ of expressions E and F is a ground substitution such that $E\sigma = F\sigma$.
   - Let F be a sentence and $\mathcal{C}$ its clause form. Then $\Box \in \mathsf{Res}^*(\mathcal{C})$ if F is satisfiable.