1. a) The only assignment that satisfies formula A is given by $\mathsf{v}(p) = \mathsf{F}$, $\mathsf{v}(q) = \mathsf{T}$, and $\mathsf{v}(r) = \mathsf{T}$.

   b) The following proof shows that formula B is valid:

1	$p \wedge q \to r$	assumption
2	p	assumption
3	q	assumption
4	$p \wedge q$	$\wedge: \mathsf{i}\ 2, 3$
5	r	$\to: \mathsf{e}\ 1, 4$
6	$q \to r$	$\to: \mathsf{i}\ 3-5$
7	$p \to (q \to r)$	$\to: \mathsf{i}\ 2-6$
8	$(p \wedge q \to r) \to (p \to (q \to r))$	$\to: \mathsf{i}\ 1-7$

2. a) ① $\forall x(\mathsf{Bird}(x) \wedge (\mathsf{Large}(\mathsf{father}(x)) \vee \mathsf{Large}(\mathsf{mother}(x))) \to \mathsf{Large}(x))$

   ② $\forall x(\mathsf{Bird}(x) \wedge \forall y\ (\mathsf{Relative}(y, x) \to \mathsf{Fly}(y)) \to \mathsf{Fly}(x))$

   ③ $\forall x(\mathsf{Bird}(x) \to \mathsf{Relative}(\mathsf{father}(x), x) \wedge \mathsf{Relative}(\mathsf{mother}(x), x))$

   ④ $\forall x(\mathsf{Bird}(x) \wedge \neg\mathsf{Eat}(x, \mathsf{worms}) \to \mathsf{Eat}(x, \mathsf{fish}))$

   ⑤ $\exists x(\mathsf{Bird}(x) \wedge \mathsf{Large}(x) \wedge \neg\mathsf{Fly}(x))$

   b) We define a suitable structure $\mathcal{A} = (A, a)$ where $A = \{f, p, w\}$ and the mapping a is defined as follows:

   - $a(\mathsf{fish}) = f$, $a(\mathsf{worms}) = w$.
   - $a(\mathsf{father}) = a(\mathsf{mother}) := f \colon A \mapsto p$, that is all function symbols are mapped to the constant function that always returns p.
   - $a(\mathsf{Bird}) = a(\mathsf{Large}) = \{p\}$, $a(\mathsf{Eat}) = \{(p, f)\}$, $a(\mathsf{Relative}) = \{(p, p)\}$, and the interpretation of all other predicates is empty.

   Then $\mathcal{A} \models F$ holds for each of the five sentences above and thus the formalisation is satisfiable.

3. a) The definition

$$\mathcal{I} \models \exists x F(x) \iff \text{there exists a variable } x \text{ such that } \mathcal{I} \models F(x)$$

   confuses the variable x with an arbitrary element of the domain. Moreover, assuming we do not distinguish between bound and free variables (as in the faulty definition) we can consider the following interpretation $\mathcal{I} = (\mathcal{A}, \ell)$ and the formula $\exists x \mathsf{P}(x) \wedge \neg P(x)$. Let the domain of $A = \{blue, green\}$, let $\mathsf{P}^{\mathcal{A}} = \{blue\}$ and let $\ell = \{x \in \mathcal{V} \mid x \mapsto green\}$. Then $\mathcal{I} \models \exists x \mathsf{P}(x) \wedge \neg P(x)$ should hold, but does not hold with respect to the faulty definition.

b) The definition missing cases for the logical symbols $\lor$, $\to$ and $\forall$. This can either corrected by adding these definitions, or by assuming that the base language only contains $\neg$, $\land$, and $\exists$ as logical symbols.

c) See Definition 3.8.

4. a) The SNF of formula C has the form

$$\forall x \forall z \forall u (\neg \mathsf{Q}(x, \mathsf{f}(x), z) \lor \mathsf{P}(\mathsf{g}(x, z, u), x, \mathsf{f}(x), u)) \ ,$$

where f, g are new Skolem functions.

b) The SNF of formula D has the form

$$\begin{aligned} \forall y \forall z \, ((\neg \mathsf{R}(\mathsf{a}, z) \lor \neg \mathsf{R}(\mathsf{a}, y) \lor \mathsf{R}(\mathsf{a}, \mathsf{f}(y, z))) \land \\ (\neg \mathsf{R}(\mathsf{a}, z) \lor \neg \mathsf{R}(\mathsf{a}, y) \lor \mathsf{R}(y, \mathsf{f}(y, z))) \land \\ (\neg \mathsf{R}(\mathsf{a}, z) \lor \neg \mathsf{R}(\mathsf{a}, y) \lor \mathsf{R}(z, \mathsf{f}(y, z)))) \ , \end{aligned}$$

where a is a new Skolem constants, f a new Skolem function.

(a) First we negate E and transform the result to obtain for example the following corresponding SNF:

$$\forall x \forall y \, ((\mathsf{R}(x) \lor \mathsf{Q}(x)) \land \neg \mathsf{R}(y) \land \neg \mathsf{Q}(\mathsf{a})) \ ,$$

where a is a new Skolem constant. We obtain $\mathcal{C} = \{\mathsf{R}(x) \lor \mathsf{Q}(x), \neg \mathsf{R}(y), \neg \mathsf{Q}(\mathsf{a})\}$ as the corresponding set of clauses. A possible resolution proof is given below, where for each inference σ denotes the most general unifier.

$$\cfrac{\cfrac{\mathsf{R}(x) \lor \mathsf{Q}(x) \quad \neg \mathsf{R}(y)}{\mathsf{Q}(x)} \ \sigma = \{y \mapsto x\} \qquad \neg \mathsf{Q}(\mathsf{a})}{\Box} \ \sigma = \{x \mapsto \mathsf{a}\}$$

5.

statement	yes	no
Consider propositional logic. Then $A_1, \ldots, A_n \models B$, asserts that $\mathsf{v}(B) = \mathsf{T}$, whenever there exists $i \in \{1, \ldots, n\}$ such that $\mathsf{v}(A_i) = \mathsf{T}$, for any assignment v.	☐	☑
Natural deduction for propositional logic is sound and complete. Furthermore it is the only formal system with these properties.	☐	☑
Let $\mathcal{A}$, $\mathcal{B}$ be first-order structures such that $\mathcal{A} \cong \mathcal{B}$. Then for every sentence F we have $\mathcal{A} \models F$ iff $\mathcal{B} \models F$.	☑	☐
If every finite subset of a set of first-order formulas $\mathcal{G}$ has a countable model, then $\mathcal{G}$ has a countable model.	☑	☐
Suppose $\mathcal{G}$ is a set of first-order formulas and $\mathcal{G} \vdash F$. Then there exists a finite subset $\mathcal{G}_0 \subseteq \mathcal{G}$ such that $\mathcal{G}_0 \vdash F$.	☑	☐
Let S be the set of satisfiable sets of first-order formulas $\mathcal{G}$. Then S fulfils the satisfaction properties.	☑	☐
Let $\mathcal{G}$ be a set of first-order formulas and let F be a first-order formula such that $\mathcal{G} \vdash F$. Then $\mathcal{G} \models \neg F$.	☐	☑
There exists a satisfiable and universal first-order sentence F (without $=$), such that F doesn't have a Herbrand model.	☐	☑
A unifier σ of expressions E and F is a ground substitution such that $E\sigma = F\sigma$.	☐	☑
Let F be a sentence and $\mathcal{C}$ its clause form. Then $\square \in \mathsf{Res}^*(\mathcal{C})$ if F is satisfiable.	☐	☑