1. a) One assignment that satisfies formula A is given by $\mathsf{v}(p) = \mathsf{T}$, $\mathsf{v}(q) = \mathsf{F}$.

   b) First we need to transform $\neg B$ into the following clause form:

$$p \qquad q \qquad \neg r \qquad \neg p \lor \neg q \lor r$$

   The following resolution proof shows that formula B is valid:

$$\cfrac{\cfrac{\cfrac{\neg p \lor \neg q \lor r \quad p}{\neg q \lor r} \quad q}{r} \quad \neg r}{\Box}$$

2. a) ① $\forall x\, (\mathsf{Dragon}(x) \land \exists y((\mathsf{Mother}(x,y) \lor \mathsf{Father}(x,y)) \land \mathsf{Happy}(y)) \to \mathsf{Happy}(x))$.

   ② $\forall x\, (\mathsf{Dragon}(x) \land \exists y\, (\mathsf{Ancestor}(x,y) \land \mathsf{Spitfire}(y)) \to \mathsf{Spitfire}(x))$.

   ③ $\forall x \forall y\, (\mathsf{Dragon}(x) \land (\mathsf{Mother}(x,y) \lor \mathsf{Father}(x,y)) \to \mathsf{Ancestor}(x,y)$

   ④ $\forall x \forall y \forall z\, (\mathsf{Ancestor}(x,y) \land \mathsf{Ancestor}(y,z) \to \mathsf{Ancestor}(x,z))$

   ⑤ $\exists x\, (\mathsf{Dragon}(x) \land \mathsf{colour}(x) = \mathsf{blue} \land \neg\mathsf{Spitfire}(x))$.

   b) We define a suitable structure $\mathcal{A} = (A, a)$ where $A = \{b, r, d\}$ and the mapping a is defined as follows:

   - $a(\mathsf{blue}) := b$, $a(\mathsf{red}) := r$.
   - $a(\mathsf{colour}) := f \colon A \mapsto b$, that is all domain elements are blue.
   - $a(\mathsf{Dragon}) = a(\mathsf{Happy}) := \{d\}$, $a(\mathsf{Mother}) = a(\mathsf{Father}) = a(\mathsf{Ancestor}) := \{(d,d)\}$, and the interpretation of all other predicates is empty. That is the only dragon in our domain is its own ancestor.

   Then $\mathcal{A} \models F$ holds for each of the five sentences above and thus the formalisation is satisfiable.

3. a) The definition is correct, the only difference to Definition 3.8 in the lecture notes, is the renaming of the variable b by x.

   NB: Note that in the *application* of the definition it has to be ensured that only formulas according to Definion 3.3 in the lecture notes are considered.

   b) The definition missing cases for the logical symbols $\lor$, $\to$ and $\forall$.

c) One possiblity is to extend the definition according to Definition 3.8 with the above replacement of x for b. Another is to restrict the first-order language (withour equality) in principle and add all other logical symbols as syntactic sugar.

4. a) The SNF of formula C has the form

$$\forall y \forall u (\mathsf{P}(\mathsf{a}) \lor \neg \mathsf{Q}(y, \mathsf{f}(y)) \lor \mathsf{R}(u, \mathsf{g}(y, u))) \ ,$$

where a, f, g are new Skolem constants or functions, respectively.

b) The SNF of formula D has the form

$$\forall x \forall z \forall w ((\neg \mathsf{P}(x) \lor \neg \mathsf{P}(z) \lor \mathsf{R}(x, w)) \land (\neg \mathsf{P}(x) \lor \neg \mathsf{Q}(x, \mathsf{f}(x)) \lor \mathsf{R}(x, w))) \ ,$$

where f is a new Skolem function.

c) First we negate E and transform the result to obtain for example the following corresponding SNF:

$$\forall x \ (\mathsf{a} = \mathsf{b} \land \mathsf{Q}(\mathsf{a}, \mathsf{c}) \land \neg \mathsf{Q}(\mathsf{b}, x)) \ ,$$

where c is a new Skolem constant. Hence we obtain the corresponding set of clauses: $\mathcal{C} = \{\mathsf{a} = \mathsf{b}, \mathsf{Q}(\mathsf{a}, \mathsf{c}), \neg \mathsf{Q}(\mathsf{b}, x)\}$. A possible paramodulation proof is given below, where σ denotes the most general unifier.

$$\dfrac{\dfrac{\mathsf{a} = \mathsf{b} \quad \mathsf{Q}(\mathsf{a}, \mathsf{c})}{\mathsf{Q}(\mathsf{b}, \mathsf{c})} \qquad \neg \mathsf{Q}(\mathsf{b}, x)}{\square} \ \sigma = \{x \mapsto \mathsf{c}\} \ .$$

5.

statement	**yes**	**no**
Consider propositional logic. Then $A_1, \dots, A_n \models B$, asserts that $\mathsf{v}(B) = \mathsf{T}$, whenever there exists $i \in \{1, \dots, n\}$ such that $\mathsf{v}(A_i) = \mathsf{T}$, for any assignment v.	☐	☑
Natural deduction for propositional logic is sound and complete.	☑	☐
Let $\mathcal{A}$, $\mathcal{B}$ be first-order structures such that $\mathcal{A} \cong \mathcal{B}$ and let ℓ be an environment. Then for every formula F we have $(\mathcal{A}, \ell) \models F$ iff $(\mathcal{B}, \ell) \models F$.	☐	☑
If formula G is obtained from formula F on replacing a subformulas A by an equivalent formula B then F and G are equivalent.	☑	☐
For any formula F there exists a formula G such that G does neither contain individual or function constants nor equality and $F \approx G$.	☑	☐
The set S of all consistent set of formulas has the satisfaction properties.	☑	☐
Let $\mathcal{G}$ be a countable set of formulas, if $\mathcal{G}$ is consistent, then $\mathcal{G}$ has only uncountable models.	☐	☑
For graphs, reachability is expressible as existential, first-order formula.	☐	☑
There exists no satisfiable set of first-order sentences $\mathcal{G}$, such that there exists no Herbrand model of $\mathcal{G}$.	☐	☑
For any first-order sentence F there exists a set of clauses $\mathcal{C} = \{C_1, \dots, C_m\}$ such that $F \approx \forall x_1 \dots \forall x_n (C_1 \land \dots \land C_m)$.	☑	☐