Problem Set 1 (for October 20)

The problems will be discussed on October 20. We use the notation of the lecture.

- Suppose $\mathcal{L}$ is a correct system such that the following two conditions hold.

  1. The set P^* is expressible in $\mathcal{L}$.
  2. For any predicate H, there is a predicate H' such that for every n, the sentence $H'(n)$ is provable in $\mathcal{L}$ iff $H(n)$ is refutable in $\mathcal{L}$.

  Show that $\mathcal{L}$ is incomplete.

- We say that a predicate H *represents* a set A in $\mathcal{L}$ if for every number n, the sentence $H(n)$ is provable in $\mathcal{L}$ iff $n \in A$.

  Suppose $\mathcal{L}$ is consistent. Show that if the set R^* is representable in $\mathcal{L}$, then $\mathcal{L}$ is incomplete.

- Let us say that a predicate H *contrarepresents* of a set A in $\mathcal{L}$ if for every number n, the sentence $H(n)$ is refutable in $\mathcal{L}$ iff $n \in A$. Show that if the P^* is contrarepresentable in $\mathcal{L}$ and $\mathcal{L}$ is consistent, then $\mathcal{L}$ is incomplete.