Problem Set 2 (for October 27)

The problems will be discussed on October 27. We use the notation of the lecture.

- For any set A of natural numbers and any function $f(x)$ (from natural numbers to natural numbers) by $f^{-1}(A)$, we mean the set of all n such that $f(n) \in A$. Prove that if A and f are Arithmetic, then so is $f^{-1}(A)$. Show the same for arithmetic.
- 1. Given two Arithmetic functions $f(x)$ and $g(y)$, show that the function $f(g(y))$ is Arithmetic.
  2. Given two Arithmetic functions $f(x)$ and $g(x, y)$, show that the functions $g(f(y), y)$, $g(x, f(y))$ and $f(g(x, y))$ are all Arithmetic.
- Let A be an infinite Arithmetic set. Then for any number y (whether in A or not), there must be an element x of A which is greater than y. Let $R(x, y)$ be the relation: x is the smallest element of A greater than y. Prove that $R(x, y)$ is Arithmetic.