	Homework
Constant	Homework
Godel's Incompleteness Theorem Georg Moser Institute of Computer Science & UIBK Winter 2011	 Property 3) in Lemma on page 101 Chapter VIII, Exercise 4. Chapter VIII, Exercise 5. Chapter VIII, Exercise 6.
tline	GM (Institute of Computer Science @ UIBK) Gödel's Incompleteness Theorem 133 Definability and Complete Representability Gödel's Incompleteness Theorem 133
Outline of the Lecture	Definability and Complete Representability Revisited
General Idea Behind Gödel's Proof abstract forms of Gödel's, Tarski's theorems, undecidable sentences of ${\cal L}$	Definition • $F(w, w)$ represents R in S if for all $(m, w) \in \mathbb{N}^n$:
Tarski's Theorem for Arithmetic the language \mathcal{L}_E , Tarski's theorem, the axiom system PE, arithmetisation of the axiom system, incompleteness of PA, Σ_1 -relations	 F(v₁,, v_n) represents R in S if for all (m₁,, m_n) ∈ Nⁿ: F(m ₁,,m_n) is provable in S ⇔ (m₁,,m_n) ∈ R F(v₁,,v_n) defines R in S if for all (m₁,,m_n) ∈ Nⁿ:
Gödel's Proof ω -consistency, Σ_0 -complete subsystems, ω -incompleteness of PA	 if R(m₁,,m_n) holds, then F(m ₁,,m_n) is provable in S if R(m₁,,m_n) is false, then F(m ₁,,m_n) is refutable in S F(v₁,,v_n) completely represents R in S if
Rosser Systems general separation principle, Rosser's undecidable sentence, Gödel and Rosser sentences compared	1 F represents R $\neg F$ represents $\sim R$ Remark
The Unprovability of Consistency definability and diagonalisation, the unprovability of consistency	if S is consistent: completely representability \Leftrightarrow definability \Rightarrow representability

xed Points and Gödel Sentences	Fixed Points and Gödel Sentences
for any expression X, $\overline{X} := \overline{\ulcorner X \urcorner}$	Gödel Sentences and Fixed Points
Definition a sentence X is called fixed point of formula $F(v)$ (in S) if $S \vdash F(\overline{X}) \leftrightarrow X$	Definition we say X is a Gödel sentence for set A with respect to S if X is provable in S iff A contains $\lceil X \rceil$
Theorem If the diagonal function d(x) is strongly definable in S, then every formula has a fixed point Proof. on the whiteboard Corollary If S is an extension of Robinson's R, then every formula has a fixed point	Definition function $f(x)$ is called acceptable in S if for every set A representable in S , $f^{-1}(A)$ is representable in S Theorem if the diagonal function $d(x)$ is acceptable in S , then for every set A representable in S , there is a Gödel sentence for A NB: if X is a Gödel sentence for a set represented by $F(v_1)$, then X is provable iff $F(\overline{X})$ is provable
Ith Predicates	Unprovability of Consistency
Truth Predicates Definition a formula $T(v_1)$ is a truth-predicate for S if for every sentence X : $S \vdash X \leftrightarrow T(\overline{X})$	Definition a formula $P(v_1)$ is called provability predicate for S if P_1 if X is provable in S , then so is $P(\overline{X})$ $P_2 \ P(\overline{X \to Y}) \to (P(\overline{X}) \to P(\overline{Y}))$ is provable in S $P_3 \ P(\overline{X}) \to P(P(\overline{X}))$ is provable in S
Theorem if S is correct, then there is no truth predicate for S	Lemma $(t \in \mathcal{D}(t_{i}))$ has a neurophility predicate, then we obtain
Theorem if S is consistent and the diagonal function is strongly definable, then there is not truth-predicate for S	let $P(v_1)$ be a provability predicate, then we obtain: P_4 if $X \to Y$ is provable, so is $P(\overline{X}) \to P(\overline{Y})$ P_5 if $X \to (Y \to Z)$ is provable, so is $P(\overline{X}) \to (P(\overline{Y}) \to P(\overline{Z}))$ P_6 if $X \to (P(\overline{X}) \to Y)$ is provable, so is $P(\overline{X}) \to P(\overline{Y})$
	Proof.

nprovability of Consistency	Abstract Form of Gödel's Second Incompleteness Theorem
Definition S is called diagonalisable if every formula has a fixed point	Abstract Form of Gödel's Second Incompleteness Theorem
Example PA is diagonalisable	Theorem if G is a fixed point of the formula $\neg P(v_1)$ and S is consistent, then G is not provable in S Proof.
Convention in the following $P(v_1)$ denotes a provability predicate	on the whiteboard
Definition	Theorem if G is a fixed point of $\neg P(v_1)$, then consis $\rightarrow G$ is provable in S
let consis := $\neg P(\overline{0} = \overline{1})$	Proof. on the whiteboard
l (Institute of Computer Science @ UIBK) Gödel's Incompleteness Theorem 140/144 stract Form of Gödel's Second Incompleteness Theorem	4 GM (Institute of Computer Science @ UIBK) Gödel's Incompleteness Theorem 141, Abstract Form of Gödel's Second Incompleteness Theorem
Theorem	To Conclude
suppose $\mathcal S$ is diagonalisable and consistent; then consis is not provable in $\mathcal S$	Discussion on FOM
Proof.	• WV: Consistency on the other hand is not an interesting problem since it has been shown by Gödel to be impossible to proof.
on the whiteboard	 HF: How would you assess the various proofs of the consistency of PA that we now have, for your purposes? []
Theorem (Löb's Theorem)	• WV: Well, that would contradict Gödel's result.
suppose S is diagonalisable; then for any sentence Y, if $P(\overline{Y}) \to Y$ is provable in S, so is Y	•
Proof. on the whiteboard	Smullyan says: Rubbish!
NB: Gödel's Second Incompleteness Theorem is a corollary to Löb's Theorem	

