Gödel's Incompleteness Theorem

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Homework

- Exercise 6 in Chapter 2, that is:

  > *By the method we have studied but using base* 10 *Gödel numbering, find a Gödel sentence X for the set of even numbers. Then X is true if the Gödel number of X is even. Is the sentence X true or false?*

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- Exercise 6 in Chapter 2, that is:

  By the method we have studied but using base 10 *Gödel numbering, find a Gödel sentence* X *for the set of even numbers. Then* X *is true if the Gödel number of* X *is even. Is the sentence* X *true or false?*

- Exercise 7 in Chapter 2, that is:

  Find an Arithmetic function $f(x)$ *such that for any number* n*, if* n *is the Gödel number of a formula* $F(v_1)$ *with just the free variable* v_1*,* $f(n)$ *is the Gödel number of a Gödel sentence for the set expressed by* $F(v_1)$*.*

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- Exercise 7 in Chapter 2, that is:

  Find an Arithmetic function $f(x)$ such that for any number n, if n is the Gödel number of a formula $F(v_1)$ with just the free variable v_1, $f(n)$ is the Gödel number of a Gödel sentence for the set expressed by $F(v_1)$.

- Exercise 1 in Chapter 3, that is:

  Let $\mathrm{Fr}(x, y)$ be the relation E_x is a variable, E_y is a formula and E_x has at least one free occurrence in E_y. Show that $\mathrm{Fr}(x, y)$ is Arithmetic.

More Homework

- Exercise 2 in Chapter 3, that is:

  > *Use the above exercise to show the following: (a) The set of Gödel numbers of sentences is Arithmetic. (b) The set of Gödel number of provable sentences in* PE *is Arithmetic.*

More Homework

- Exercise 2 in Chapter 3, that is:

  Use the above exercise to show the following: (a) The set of Gödel numbers of sentences is Arithmetic. (b) The set of Gödel number of provable sentences in PE *is Arithmetic.*

- Exercise 3 in Chapter 3, that is

  Given any finite sequence $(a_1, b_1), (a_2, b_2), \ldots, (a_n, b_n)$ *of ordered pairs of numbers in* K_{11}*, we assign the sequence number*

  $$\delta\delta a_1 \delta b_1 \delta\delta \cdots \delta\delta a_n \delta b_n \delta\delta$$

  We let $\mathrm{Seq}_2(x)$ *denote that* x *is sequence number. We let* $(x, y) \in z$ *denote that the pair* (x, y) *is a member of the sequence, numbered by* z*. Finally let* $(x_1, y_1) \prec_z (x_2, y_2)$ *denote that* (x_1, y_1) *occurs in* z *before* (x_2, y_2)*.*

And Even More Homework

- Exercise 4 in Chapter 3, that is:

  [...] Now let $Sub(E, w, t, F)$ *be the relation "E is a term or formula, w is a variable, t is a term and* $F = E\{w \mapsto t\}$*". Let* $sub(x_1, x_2, x_3, x_4)$ *be the corresponding relation on Gödel numbers. [...] Show that* $sub(x_1, x_2, x_3, x_4)$ *is Arithmetic.*