Computational Logic

Gödel's Incompleteness Theorem

Georg Moser

Institute of Computer Science @ UIBK

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Homework

- Exercise 6 in Chapter 2, that is:

  By the method we have studied but using base 10 Gödel numbering, find a Gödel sentence X for the set of even numbers. Then X is true if the Gödel number of X is even. Is the sentence X true or false?

- Exercise 7 in Chapter 2, that is:

  Find an Arithmetic function $f(x)$ such that for any number n, if n is the Gödel number of a formula $F(v_1)$ with just the free variable v_1, $f(n)$ is the Gödel number of a Gödel sentence for the set expressed by $F(v_1)$.

- Exercise 1 in Chapter 3, that is:

  Let $\mathrm{Fr}(x, y)$ be the relation E_x is a variable, E_y is a formula and E_x has at least one free occurrence in E_y. Show that $\mathrm{Fr}(x, y)$ is Arithmetic.

More Homework

- Exercise 2 in Chapter 3, that is:

  Use the above exercise to show the following: (a) The set of Gödel numbers of sentences is Arithmetic. (b) The set of Gödel number of provable sentences in PE *is Arithmetic.*

- Exercise 3 in Chapter 3, that is

  Given any finite sequence $(a_1, b_1), (a_2, b_2), \ldots, (a_n, b_n)$ of ordered pairs of numbers in K_{11}, we assign the sequence number

  $$\delta\delta a_1 \delta b_1 \delta\delta \cdots \delta\delta a_n \delta b_n \delta\delta$$

  We let $\mathrm{Seq}_2(x)$ denote that x is sequence number. We let $(x, y) \in z$ denote that the pair (x, y) is a member of the sequence, numbered by z. Finally let $(x_1, y_1) \prec_z (x_2, y_2)$ denote that (x_1, y_1) occurs in z before (x_2, y_2).

And Even More Homework

- Exercise 4 in Chapter 3, that is:

  [...] Now let $\mathrm{Sub}(E, w, t, F)$ be the relation "E is a term or formula, w is a variable, t is a term and $F = E\{w \mapsto t\}$". Let $\mathrm{sub}(x_1, x_2, x_3, x_4)$ be the corresponding relation on Gödel numbers. [...] Show that $\mathrm{sub}(x_1, x_2, x_3, x_4)$ is Arithmetic.