Introduction to Model Checking

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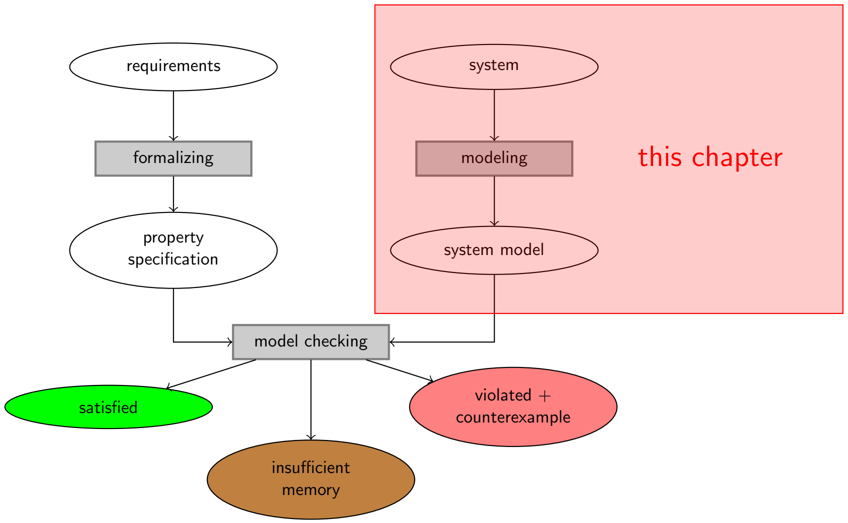
Outline

Model checking overview



Motivation

- so far, input to model checker is transition system and formula
- for modeling want higher-level description as transition system
  - use variables

  ⇒ program graphs
  - use communication

  ⇒ channel systems
  - use textual format

  ⇒ Promela

Outline

- Program Graphs
- Channel systems
- Promela
  - Promela - Syntax and Intuitive Meaning
  - Formal semantics
- The State-Space Explosion Problem

Beverage vending machine revisited

transitions (with variables, conditions, and actions):

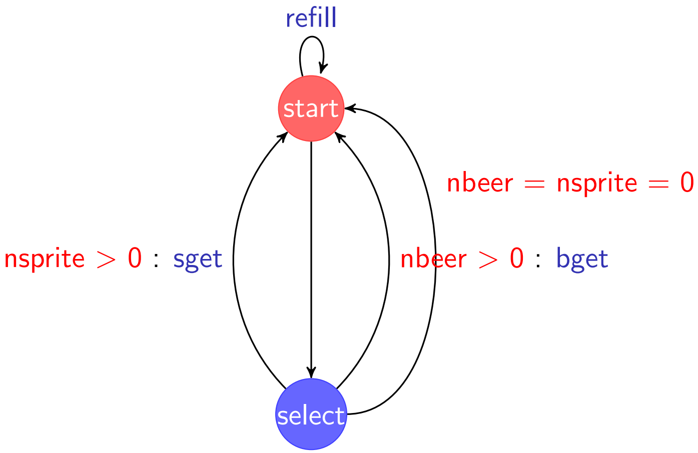
$$start \longrightarrow select \qquad start \xrightarrow{refill} start$$

$$select \xrightarrow{nsprite>0:sget} start \qquad select \xrightarrow{nbeer>0:bget} start$$

$$select \xrightarrow{nsprite=0 \wedge nbeer=0} start$$

Action	Effect on variables
sget	$nsprite := nsprite - 1$
bget	$nbeer := nbeer - 1$
refill	$nsprite := max;\ nbeer := max$

Program graph representation



Some preliminaries

- typed variables *Var* with evaluation η that assigns values of domain D to variables
  - e.g., $\eta(x) = 17$ and $\eta(y) = green$
- the set of Boolean conditions over *Var*
  - propositional logic formulas whose propositions are of the form "$\overline{x} \in \overline{D}$"
  - $(nsprite \geqslant 1)\ \wedge\ (y = yellow)\ \wedge\ (x \leqslant 2{\cdot}x')$
- effect of the actions is formalized by means of a mapping:

$$Effect : Act \times Eval(Var) \rightarrow Eval(Var)$$

  - e.g., for action α use update $x := y == yellow\ ?\ 2 \cdot x : x - 1$, and evaluation η is given by $\eta(x) = 17$ and $\eta(y) = red$
  - $Effect(\alpha, \eta)(x) \ = \ \eta(x) - 1 = 16$, and $Effect(\alpha, \eta)(y) \ = \eta(y) = red$

Program graphs

a program graph PG over set Var of typed variables is tuple

$$(Loc, Act, Effect, \longrightarrow, Loc_0, g_0) \quad \text{where}$$

- Loc is a set of locations with initial locations $Loc_0 \subseteq Loc$
- Act is a set of actions including the empty action ␣, usually not written
- $Effect : Act \times Eval(Var) \to Eval(Var)$ is the effect function
  - ␣ has never an effect, i.e., $Effect(␣, \eta) = \eta$
- $\longrightarrow \subseteq Loc \times (Cond(Var) \times Act) \times Loc$, transition relation
  - $Cond(Var)$: Boolean conditions over Var true is not written
- $g_0 \in Cond(Var)$ is the initial condition

notation: $\ell \xrightarrow{g:\alpha} \ell'$ denotes $(\ell, g, \alpha, \ell') \in \longrightarrow$

Beverage vending machine

NOT IN HANDOUT, GRAPH IN PRESENTATION

- $Loc = \{ start, select \}$ with $Loc_0 = \{ start \}$
- $Act = \{ bget, sget, refill \}$
- $Var = \{ nsprite, \ nbeer \}$ with domain $\{ 0, 1, \ldots, max \}$
- $$\begin{aligned} Effect(sget, \eta) &= \eta[nsprite := nsprite - 1] \\ Effect(bget, \eta) &= \eta[nbeer := nbeer - 1] \\ Effect(refill, \eta) &= [nsprite := max, nbeer := max] \end{aligned}$$
- $g_0 = (nsprite = max \wedge nbeer = max)$

From program graphs to transition systems

- basic strategy: unfolding
  - state = location ℓ + evaluation η
  - initial state = initial location satisfying the initial condition g_0
- propositions and labeling
  - propositions: "at ℓ" and "$x \in D$" for $D \subseteq dom(x)$
  - $\langle \ell, \eta \rangle$ is labeled with "at ℓ" and all conditions that hold in η
- if $\ell \xrightarrow{g:\alpha} \ell'$ and g holds in η then $\langle \ell, \eta \rangle \to \langle \ell', Effect(\alpha, \eta) \rangle$

Structured operational semantics

- notation $\dfrac{\text{premise}}{\text{conclusion}}$ means:
  if the proposition above the "solid line" (i.e., the premise) holds, then the proposition under the fraction bar (i.e., the conclusion) holds
- such "if ... then ..." propositions are also called inference rules
- if the premise is a tautology, it may be omitted
- in the latter case, the rule is also called an axiom

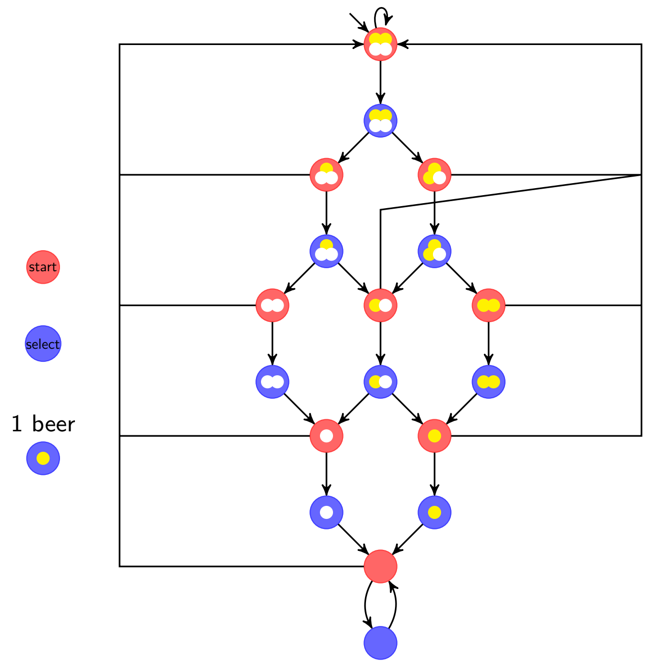
Transition systems for program graphs

the transition system $TS(PG)$ of program graph

$$PG = (Loc, Act, Effect, \longrightarrow, Loc_0, g_0)$$

over set Var of variables is the tuple $(S, \longrightarrow, I, AP, L)$ where

- $S = Loc \times Eval(Var)$
- $\longrightarrow \subseteq S \times S$ is defined by the rule: $\dfrac{\ell \xrightarrow{g:\alpha} \ell' \quad \wedge \quad \eta \models g}{\langle \ell, \eta \rangle \rightarrow \langle \ell', Effect(\alpha, \eta) \rangle}$
- $I = \{\langle \ell, \eta \rangle \mid \ell \in Loc_0, \eta \models g_0\}$
- $AP = Loc \cup Cond(Var)$ and $L(\langle \ell, \eta \rangle) = \{\ell\} \cup \{g \in Cond(Var) \mid \eta \models g\}$



Outline

Concurrent systems

- program graphs
  - suited for modeling sequential data-dependent systems
- what about concurrent systems?
  - threading
  - distributed algorithms and communication protocols
- can we model:
  - synchronous communication?
  - asynchronous communication?

Interleaving

- construct concurrent system from several (sequential) components
- actions of independent components are merged or interleaved
  - a single or more processors are available (perhaps on different computers)
  - on which the actions of the processes are interlocked
- no assumptions on the order of processes
  - possible orders for independent processes P and Q:

$$\begin{array}{cccccccccc} P & Q & P & Q & P & Q & Q & Q & P & \dots \\ P & P & Q & P & P & Q & P & P & Q & \dots \\ P & Q & P & P & Q & P & P & P & Q & \dots \end{array}$$

- justification for interleaving:
  - the effect of concurrently executed, independent actions α and β equals the effect when α and β are successively executed in arbitrary order

  P: x++;... Q: y++ ..., parallel execution = sequential execution

Channels

usually, processes exchange data in some way $\Rightarrow$ channels

- processes communicate via channels ($c \in \mathit{Chan}$)
- channels are first-in, first-out buffers
- channels are typed (wrt. their content — $\mathit{dom}(c)$)
- channels buffer messages (of appropriate type)
- channel capacity = maximum # messages that can be stored
  - c is a channel with finite capacity $\mathit{cap}(c)$
  - if $\mathit{cap}(c) > 0$, there is some "delay" between sending and receiving
  - if $\mathit{cap}(c) = 0$, then communication via c amounts to handshaking

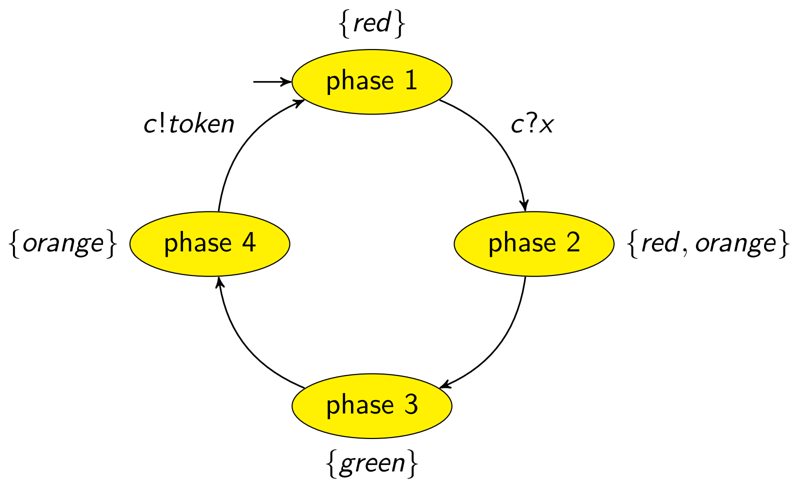
Channels

- process P_i = program graph PG_i + communication actions

  $c!v$ transmit the value v along channel c

  $c?x$ receive message via channel c and assign it to variable x
- Comm =
  $\{\ c!v,\ c?x \mid c \in \mathit{Chan},\ v \in \mathit{dom}(c),\ x \in \mathit{Var}.\ \ \mathit{dom}(x) \supseteq \mathit{dom}(c)\ \}$
- sending and receiving a message
  - $c!v$ puts the value v at the rear of the buffer c (if c is not full)
  - $c?x$ retrieves the front element of the buffer and assigns it to x (if c is not empty)
  - if $\mathit{cap}(c) = 0$, channel c has no buffer
  - if $\mathit{cap}(c) = 0$, sending and receiving can takes place simultaneously this is called synchronous message passing or handshaking
  - if $\mathit{cap}(c) > 0$, sending and receiving can never take place simultaneously this is called asynchronous message passing

Example: Traffic Light



say that channel only has one value: *token*, say that $\mathit{cap}(c) = 0$

Channel systems

a program graph over $(Var, Chan)$ is a tuple

$$PG = (Loc, Act, Effect, \rightarrow, Loc_0, g_0)$$

where

$$\rightarrow \;\subseteq\; Loc \times (Cond(Var) \times Act) \times Loc \;\cup\; \underbrace{Loc \times (Cond(Var) \times Comm) \times Loc}_{\text{communication actions}}$$

a channel system CS over $(\bigcup_{0<i\leqslant n} Var_i, Chan)$:

$$CS \;=\; [PG_1 \mid \cdots \mid PG_n]$$

with program graphs PG_i over $(Var_i, Chan)$

Example: Traffic lights

NOT IN HANDOUT, GRAPH IN PRESENTATION

$$Crossing \;=\; [TrafficLight \mid TrafficLight \mid Starter]$$

- only two traffic lights $\Rightarrow$ both wait for input infinitely long
- therefore use additional “starter” to send one input

Communication actions

Handshaking

- if $cap(c) = 0$, then process P_i can perform $\ell_i \xrightarrow{g:c!v} \ell_i'$ only
- ... if P_j, say, can perform $\ell_j \xrightarrow{g':c?x} \ell_j'$ and ...
- if both g and g' are satisfied, and
- the effect corresponds to the (atomic) distributed assignment $x := v$.

Asynchronous message passing
later ...

Transition system semantics of a channel system

let $CS = [PG_1 \mid \cdots \mid PG_n]$ be a channel system over $(Chan, Var)$ with

$$PG_i = (Loc_i, Act_i, Effect_i, \rightarrow_i, Loc_{0,i}, g_{0,i}), \qquad \text{for } 0 < i \leqslant n$$

$TS(CS)$ is the transition system $(S, \rightarrow, I, AP, L)$ where:

- $S \;=\; (Loc_1 \times \cdots \times Loc_n) \times Eval(Var) \times Eval(Chan)$
- $\rightarrow$ is defined by the inference rules on the next slides
- $I = \left\{ \langle \ell_1, \ldots, \ell_n, \eta, \xi_0 \rangle \;\middle|\; \forall i.\ (\ell_i \in Loc_{0,i} \wedge \eta \models g_{0,i}) \wedge \forall c.\ \xi_0(c) = \varepsilon \right\}$
- $AP \;= \biguplus_{0<i\leqslant n} Loc_i \;\uplus\; Cond(Var)$
- $L(\langle \ell_1, \ldots, \ell_n, \eta, \xi \rangle) \;=\; \{\, \ell_1, \ldots, \ell_n \,\} \;\cup\; \{\, g \in Cond(Var) \mid \eta \models g \,\}$

Inference rules (I)

- interleaving for $\alpha \in Act_i$:

$$\frac{\ell_i \xrightarrow{g:\alpha}_i \ell_i' \quad \wedge \quad \eta \models g}{\langle \ell_1, \ldots, \ell_i, \ldots, \ell_n, \eta, \xi \rangle \rightarrow \langle \ell_1, \ldots, \ell_i', \ldots, \ell_n, \eta', \xi \rangle}$$

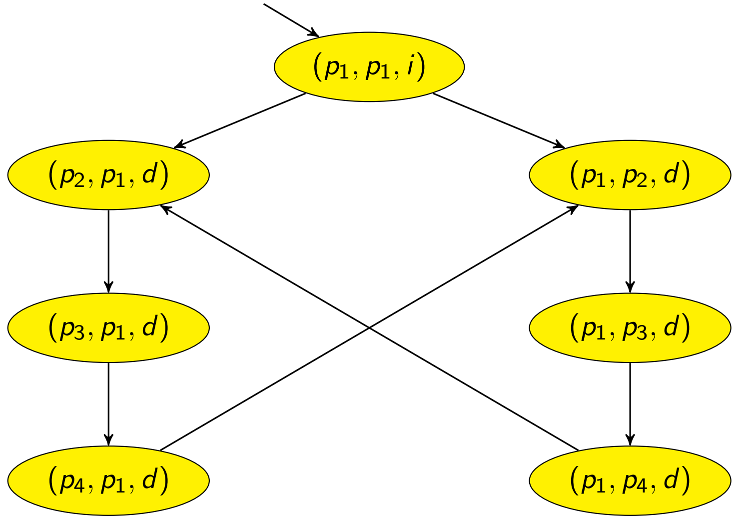
  where $\eta' = \mathit{Effect}(\alpha, \eta)$
- synchronous message passing over $c \in \mathit{Chan}$, $\mathit{cap}(c) = 0$:

$$\frac{\ell_i \xrightarrow{g:c?x}_i \ell_i' \quad \wedge \quad \ell_j \xrightarrow{g':c!v}_j \ell_j' \quad \wedge \quad (\eta \models g \wedge g') \quad \wedge \quad i \neq j}{\langle \ell_1, \ldots, \ell_i, \ldots, \ell_j, \ldots, \ell_n, \eta, \xi \rangle \rightarrow \langle \ell_1, \ldots, \ell_i', \ldots, \ell_j', \ldots, \ell_n, \eta', \xi \rangle}$$

  where $\eta' = \eta[x := v]$

Example: Traffic lights

NOT IN HANDOUT, PROGRAM GRAPH IN PRESENTATION



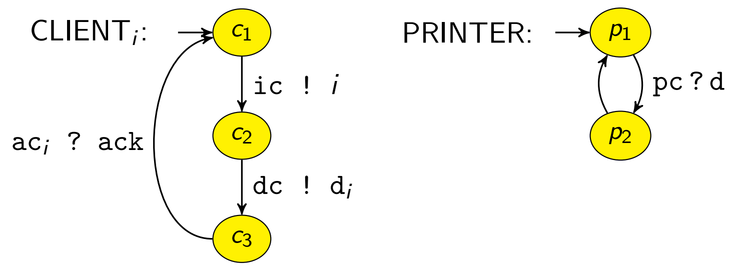
mention unreachable states, no evaluation, no channel evaluations

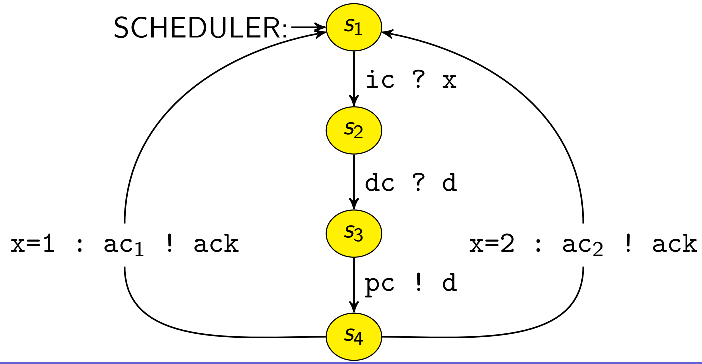
Example protocol

- two clients, one scheduler, one printer
- clients send data to scheduler which sends this data further to printer
- before sending data, clients have to initialize connection by sending id
- after data has been delivered by printer, scheduler sends ack. to client
- scheduler should always be able to receive data

$\Rightarrow$ proper modeling requires asynchronous message passing
  - if $\mathit{cap}(c) > 0$, then process P_i can perform $\ell_i \xrightarrow{g:c!v} \ell_i'$
  - … iff g is satisfied and less than $\mathit{cap}(c)$ messages are stored in c
  - P_j may perform $\ell_j \xrightarrow{g:c?x} \ell_j'$ iff g is satisfied and c is not empty
  - then the first element v of the buffer is extracted and assigned to x (atomically)

	executable if …	effect
$g : c!v$	g is sat. and c is not full	$\mathit{Enqueue}(c, v)$
$g : c?x$	g is sat. and c is not empty	$x := \mathit{Front}(c)\,;\ \mathit{Dequeue}(c)$;





Channel evaluations

- a channel evaluation ξ is
  - a mapping from channel $c \in Chan$ onto a sequence $\xi(c) \in dom(c)^*$ such that
  - current length cannot exceed the capacity of c: $len(\xi(c)) \leqslant cap(c)$
  - $\xi(c) = v_1\, v_2 \ldots v_k$ ($cap(c) \geqslant k$) denotes v_1 is at front of buffer etc.
- $\xi[c := v_1 \ldots v_k]$ denotes the channel evaluation

$$\xi[c := v_1 \ldots v_k](c') = \begin{cases} \xi(c') & \text{if } c \neq c' \\ v_1 \ldots v_k & \text{if } c = c' \end{cases}$$

- initial channel evaluation ξ_0 equals $\xi_0(c) = \varepsilon$ for any c

Inference rules (II)

asynchronous message passing for $c \in Chan$, $cap(c) > 0$:

- receive a value along channel c and assign it to variable x:

$$\frac{\ell_i \xrightarrow{g:c?x}_i \ell_i' \;\wedge\; \xi(c) = v_1 \ldots v_k \;\wedge\; k > 0 \;\wedge\; \eta \models g}{\langle \ell_1, \ldots, \ell_i, \ldots, \ell_n, \eta, \xi\rangle \to \langle \ell_1, \ldots, \ell_i', \ldots, \ell_n, \eta', \xi'\rangle}$$

  where $\eta' = \eta[x := v_1]$ and $\xi' = \xi[c := v_2 \ldots v_k]$

- transmit value $v \in dom(c)$ over channel c:

$$\frac{\ell_i \xrightarrow{g:c!v}_i \ell_i' \;\wedge\; \xi(c) = v_1 \ldots v_k \;\wedge\; k < cap(c) \;\wedge\; \eta \models g}{\langle \ell_1, \ldots, \ell_i, \ldots, \ell_n, \eta, \xi\rangle \to \langle \ell_1, \ldots, \ell_i', \ldots, \ell_n, \eta, \xi'\rangle}$$

  where $\xi' = \xi[c := v_1\, v_2 \ldots v_k\, v]$

Transition system of example protocol

let $c(ac_1) = c(ac_2) = c(pc) = 0$ and $c(ic) = c(dc) > 0$
CHANNEL SYSTEM SHOWN IN PRESENTATION

CL_1	CL_2	SC	PR	x	$SC.d$	$PR.d$	ic	dc
c_1	c_1	s_1	p_1	?	?	?	ϵ	ϵ
c_2	c_1	s_1	p_1	?	?	?	1	ϵ
c_2	c_1	s_2	p_1	1	?	?	ϵ	ϵ
c_2	c_2	s_2	p_1	1	?	?	2	ϵ
c_2	c_3	s_2	p_1	1	?	?	2	d_2
c_2	c_3	s_3	p_1	1	d_2	?	2	ϵ
c_2	c_3	s_4	p_2	1	d_2	d_2	2	ϵ
c_3	c_3	s_4	p_2	1	d_2	d_2	2	d_1
c_1	c_3	s_1	p_2	1	d_2	d_2	2	d_1

problem: client 1 gets acknowledge although data 2 is send to printer

Outline

nanoPromela

- Promela (Process Meta Language) is modeling language for SPIN
  - most widely used model checker SPIN
  - developed by Gerard Holzmann (Bell Labs, NASA JPL)
  - ACM Software Award 2002
- nanoPromela is the core of Promela
  - shared variables and channel-based communication
  - formal semantics of a Promela model is a channel system
  - processes are defined by means of a guarded command language
- no explicit actions, statements describe effect of actions

nanoPromela

nanoPromela-program $\overline{\mathcal{P}} = [\mathcal{P}_1 | \ldots | \mathcal{P}_n]$ with $\mathcal{P}_i$ processes
a process is specified by a statement:

$$
\begin{array}{lll}
\text{stmt} & ::= & \textbf{skip} \mid x := \text{expr} \mid c?x \mid c!\text{expr} \mid \\
& & \text{stmt}_1 \,;\, \text{stmt}_2 \mid \textbf{atomic}\{\text{assignments}\} \mid \\
& & \textbf{if} \;\; :: g_1 \Rightarrow \text{stmt}_1 \;\ldots\; :: g_n \Rightarrow \text{stmt}_n \;\; \textbf{fi} \mid \\
& & \textbf{do} \;\; :: g_1 \Rightarrow \text{stmt}_1 \;\ldots\; :: g_n \Rightarrow \text{stmt}_n \;\; \textbf{od} \\
\text{assignments} & ::= & x_1 := \text{expr}_1 \,;\, x_2 := \text{expr}_2 \,;\, \ldots \,;\, x_m := \text{expr}_m
\end{array}
$$

- x is a variable in Var, expr an expression and c a channel, g_i a guard
- assume the Promela specification is type-consistent

Conditional statements

$$\textbf{if} \;\; :: g_1 \Rightarrow \text{stmt}_1 \ldots :: g_n \Rightarrow \text{stmt}_n \;\textbf{fi}$$

- nondeterministic choice between statements stmt_i for which g_i holds
- test-and-set semantics: (deviation from Promela)
  - guard evaluation + selection of enabled command + execution first atomic step of selected statement is all performed atomically
- **if**–**fi**–command blocks if no guard holds
  - parallel processes may unblock a process by changing shared variables
  - e.g., when y=0, **if** $:: y > 0 \Rightarrow x := 42$ **fi** waits until y exceeds 0
- standard abbreviations:
  - **if** g **then** stmt_1 **else** stmt_2 **fi** $\equiv$ **if** $:: g \Rightarrow \text{stmt}_1 \;\; :: \neg g \Rightarrow \text{stmt}_2$ **fi**
  - **if** g **then** stmt_1 **fi** $\equiv$ **if** $:: g \Rightarrow \text{stmt}_1 \;\; :: \neg g \Rightarrow$ **skip** **fi**

Iteration statements

$$\textbf{do} \;\; :: g_1 \Rightarrow \text{stmt}_1 \ldots :: g_n \Rightarrow \text{stmt}_n \;\textbf{od}$$

- iterative execution of nondeterministic choice among $g_i \Rightarrow \text{stmt}_i$
  - where guard g_i holds in the current state
- no blocking if all guards are violated; instead, loop is aborted
- **while** g **do** stmt **od** $\equiv$ **do** $:: g \Rightarrow$ stmt **od**
- no break-statements to abort a loop

Beverage vending machine

the following nanoPromela program describes its behaviour:

```
atomic { nbeer := max; nsprite := max };
do :: true => skip; // insert coin
              if :: nsprite > 0 => nsprite := nsprite - 1
                 :: nbeer   > 0 => nbeer   := nbeer   - 1
                 :: nsprite = nbeer = 0 => skip
              fi
   :: true => atomic { nbeer := max; nsprite := max }
od
```


Client-scheduler-printer example

```
------- CLIENT i ------------
do :: true => ic ! i;
              dc ! d_i;
              ac_i ? ack
od
------- SCHEDULER -----------
do :: true => ic ? x;
              dc ? d;
              pc ! d;
              if :: x = 1 => ac_1 ! ack
                 :: x = 2 => ac_2 ! ack
              fi
od
------- PRINTER -------------
do :: true => pc ? d; skip
od
```


Formal semantics

the semantics of a nanoPromela-statement over $(Var, Chan)$ is a program graph over $(Var, Chan)$.

the program graphs $PG_1, \ldots, PG_n$ for the processes $\mathcal{P}_1, \ldots, \mathcal{P}_n$ of a nanoPromela-program $\overline{\mathcal{P}} = [\mathcal{P}_1 | \ldots | \mathcal{P}_n]$ constitute a channel system over $(Var, Chan)$

the locations of the program graph PG_i are the sub-statements of the nanoPromela-program $\mathcal{P}_i$

Sub-statements

for statement stmt its sub-statements $\mathcal{S}ub(\text{stmt})$ is smallest set of statements such that

- $\textbf{exit} \in \mathcal{S}ub(\text{stmt})$
- $\text{stmt} \in \mathcal{S}ub(\text{stmt})$
- if $\text{stmt}' \in \mathcal{S}ub(\text{stmt})$ then $\mathcal{S}ub(\text{stmt}') \subseteq \mathcal{S}ub(\text{stmt})$
- if $\text{stmt}' \in \mathcal{S}ub(\text{stmt}_1)$ then $\text{stmt}'; \text{stmt}_2 \in \mathcal{S}ub(\text{stmt}_1; \text{stmt}_2)$
- if $\text{stmt}' \in \mathcal{S}ub(\text{stmt}_2)$ then $\text{stmt}' \in \mathcal{S}ub(\text{stmt}_1; \text{stmt}_2)$
- if $\text{stmt}' \in \mathcal{S}ub(\text{stmt}_i)$ then $\text{stmt}' \in \mathcal{S}ub(\textbf{if} \ldots :: g_i \Rightarrow \text{stmt}_i \ldots \textbf{fi})$
- if $\text{stmt}' \in \mathcal{S}ub(\text{stmt}_i)$ then $\text{stmt}'; \text{loop} \in \mathcal{S}ub(\text{loop})$ where $\text{loop} = \textbf{do} \ldots :: g_i \Rightarrow \text{stmt}_i \ldots \textbf{od}$

Inference rules

$$\frac{}{\textbf{skip} \rightarrow \textbf{exit}}$$

$$\frac{}{x := \text{expr} \xrightarrow{\text{assign}(x,\text{expr})} \textbf{exit}}$$

assign(x, expr) denotes the action that only changes x, no other variables

$$\frac{}{c?x \xrightarrow{c?x} \textbf{exit}} \qquad \frac{}{c!\text{expr} \xrightarrow{c!\text{expr}} \textbf{exit}}$$

Inference rules

$$\frac{}{\textbf{atomic}\{x_1 := \text{expr}_1; \dots; x_m := \text{expr}_m\} \xrightarrow{\alpha_m} \textbf{exit}}$$

where $\alpha_0 = id$, $\alpha_i = \mathit{Effect}(\text{assign}(x_i, \text{expr}_i), \mathit{Effect}(\alpha_{i-1}, \eta))$ for $1 \leqslant i \leqslant m$

$$\frac{\text{stmt}_1 \xrightarrow{g:\alpha} \text{stmt}_1' \neq \textbf{exit}}{\text{stmt}_1; \text{stmt}_2 \xrightarrow{g:\alpha} \text{stmt}_1'; \text{stmt}_2}$$

$$\frac{\text{stmt}_1 \xrightarrow{g:\alpha} \textbf{exit}}{\text{stmt}_1; \text{stmt}_2 \xrightarrow{g:\alpha} \text{stmt}_2}$$

Inference rules

$$\frac{\text{stmt}_i \xrightarrow{h:\alpha} \text{stmt}_i'}{\textbf{if} \dots :: g_i \Rightarrow \text{stmt}_i \dots \textbf{fi} \xrightarrow{g_i \wedge h:\alpha} \text{stmt}_i'}$$

$$\frac{\text{stmt}_i \xrightarrow{h:\alpha} \text{stmt}_i' \neq \textbf{exit}}{\textbf{do} \dots :: g_i \Rightarrow \mathit{stmt}_i \dots \textbf{od} \xrightarrow{g_i \wedge h:\alpha} \text{stmt}_i'; \textbf{do} \dots \textbf{od}}$$

$$\frac{\text{stmt}_i \xrightarrow{h:\alpha} \textbf{exit}}{\textbf{do} \dots :: g_i \Rightarrow \mathit{stmt}_i \dots \textbf{od} \xrightarrow{g_i \wedge h:\alpha} \textbf{do} \dots \textbf{od}}$$

$$\frac{}{\textbf{do} \dots :: g_i \Rightarrow \mathit{stmt}_i \dots \textbf{od} \xrightarrow{\neg g_1 \wedge \dots \wedge \neg g_n} \textbf{exit}}$$

Example: one step

let *loop* be a shortcut for

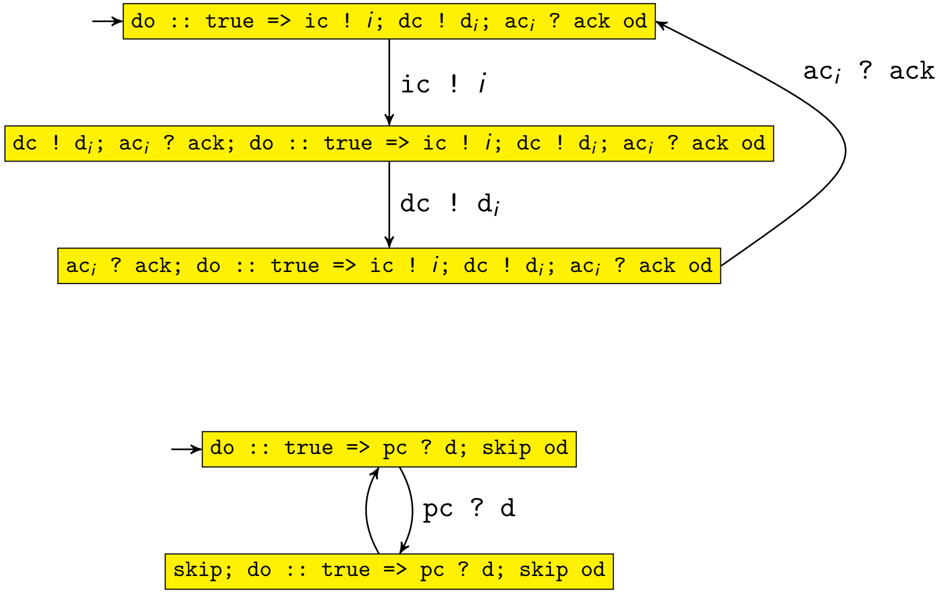
$$\texttt{do :: true => ic ! } i\texttt{; dc ! d}_i\texttt{; ac}_i\texttt{ ? ack od}$$

derive the following step in the program graph of the client

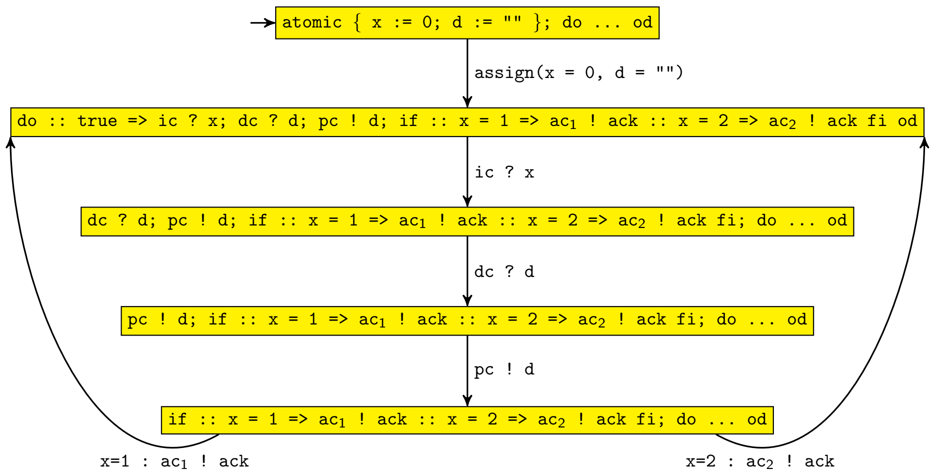
$$\cfrac{\cfrac{\cfrac{}{\texttt{ic ! } i \xrightarrow{\texttt{ic ! } i} \texttt{exit}}}{\texttt{ic ! } i\texttt{; dc ! d}_i\texttt{; ac}_i\texttt{ ? ack} \xrightarrow{\texttt{ic ! } i} \texttt{dc ! d}_i\texttt{; ac}_i\texttt{ ? ack}}}{\mathit{loop} \xrightarrow{\texttt{ic ! } i} \texttt{dc ! d}_i\texttt{; ac}_i\texttt{ ? ack; } \mathit{loop}}$$

construct by going left-down, left-up, right-up, right-down

Example: client and printer



Example: scheduler



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  - Promela - Syntax and Intuitive Meaning
  - Formal semantics
- The State-Space Explosion Problem

Sequential programs

- # states of a simple program graph is:

$$\left|\#\text{program locations}\right| \cdot \prod_{\textit{variable } x} \left| dom(x) \right|$$

$\Rightarrow$ number of states grows exponentially in number of program variables
  - N variables with k possible values each yields k^N states
  - this is called the state-space explosion problem
- program with 10 locations, 3 bools, 4 integers (in range $0 \ldots 9$):

$$10 \cdot 2^3 \cdot 10^4 = 800,000 \text{ states}$$

- adding a single 50-positions bit-array yields $800,000 \cdot 2^{50}$ states

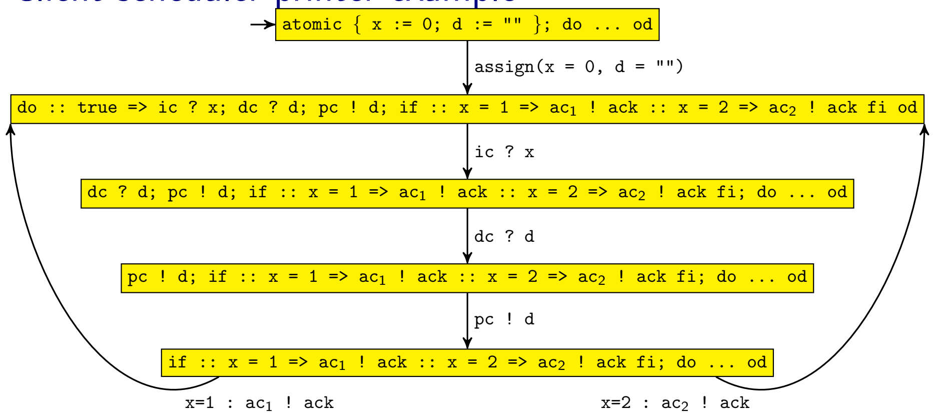
Channel systems

- each channel c has a bounded capacity $cap(c)$ and a domain $dom(c)$
- \# states of system with N components and K channels is:

$$\prod_{i=1}^{N}\left(|\#\text{program locations}| \prod_{\textit{variable } x} |\,dom(x)|\right)\cdot\prod_{j=1}^{K}|dom(c_j)|^{cap(c_j)+1}$$

this is the underlying structure of Promela

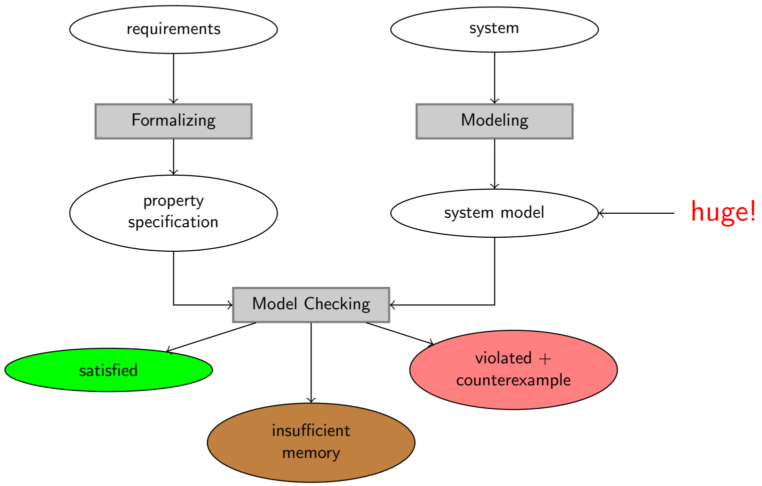
Client-scheduler-printer example



for channel capacity 6 and binary data obtain

$$\underbrace{3}_{\text{client 1}}\cdot\underbrace{3}_{\text{client 2}}\cdot\underbrace{5\cdot 2^2}_{\text{scheduler}}\cdot\underbrace{2\cdot 2}_{\text{printer}}\cdot\underbrace{2^{6+1}}_{ic}\cdot\underbrace{2^{6+1}}_{dc} = 45\cdot 2^{18} = 11,796,480 \text{ states}$$

The Need for Automated Verification



Summary of Modeling Concurrent Systems

- transition systems are fundamental for modeling software should be generated from high-level modeling language
- program graphs = states with variables
- interleaving = execution of independent concurrent processes by nondeterminism
- channel systems = program graphs + first-in first-out communication
  - handshaking for channels of capacity 0
  - asynchronous message passing when capacity exceeds 0
  - semantical model of Promela
- formal semantic for Promela ⇒ know exactly which system is verified
- size of transition systems grows exponentially
  - in the number of concurrent components, number of variables, and size of channels