Introduction to Model Checking (VO) WS 2008/2009 LVA 703503

First name: _____

Last name: _____

Matriculation number: _____

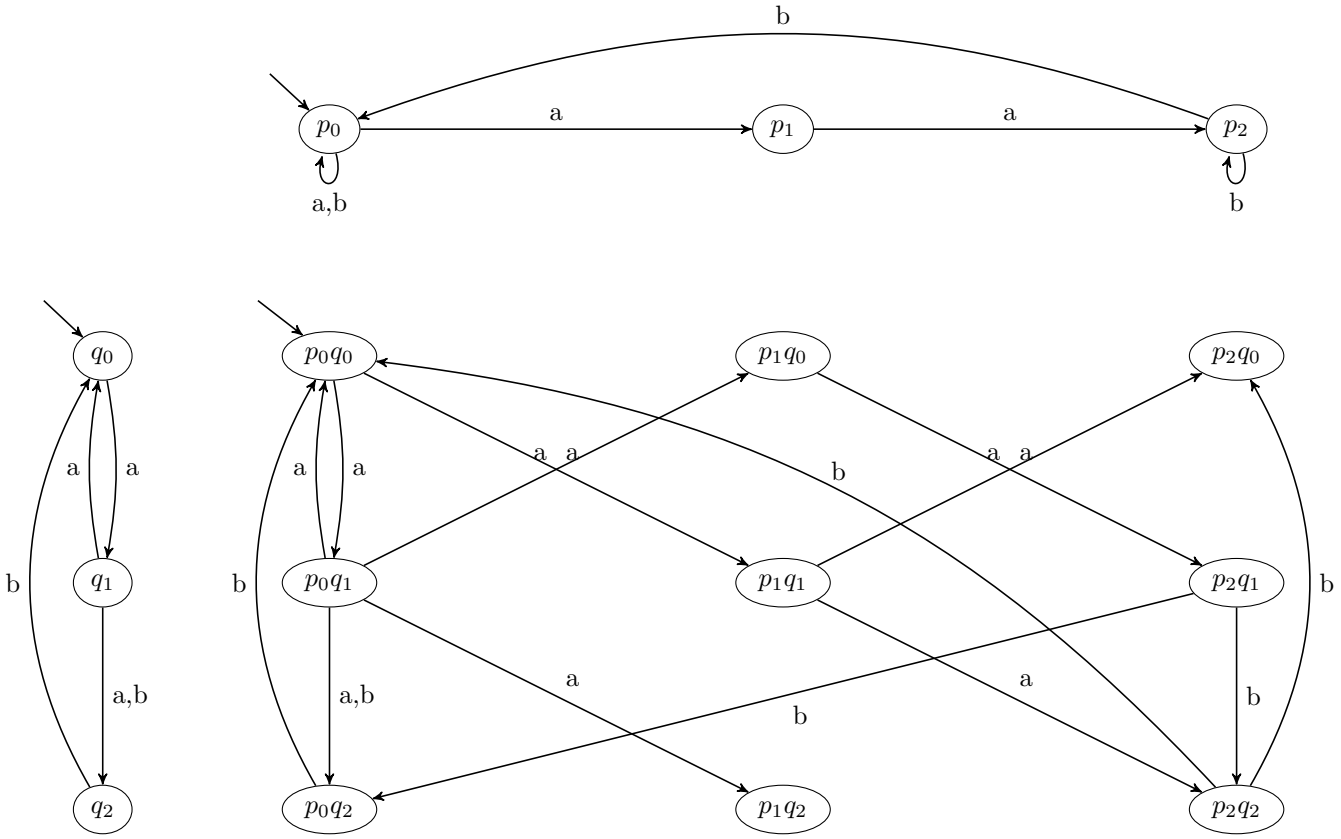
- Please answer all exercises in a readable and precise way.
- Please cross out solution attempts which are replaced by another solution.
- Please do not remove the staples of the exam.
- Cheating is not allowed. Everyone who is caught will fail the exam.

Exercise	Maximal points	Points
1	**21**	
2	**20**	
3	**20**	
4	**9**	
Σ	**70**	
Grade		

Exercise 1 (14+5+2 points)

Consider the GNBAs $\mathcal{A}_1 = (\{p_0, p_1, p_2\}, \Sigma, p_0, \delta_1, \{p_0, p_2\}, \{p_1\})$ and $\mathcal{A}_2 = (\{q_0, q_1, q_2\}, \Sigma, q_0, \delta_2, \{q_0, q_1\})$.

(i) Construct the GNBA $\mathcal{A}$ for the intersection of $\mathcal{A}_1$ and $\mathcal{A}_2$.



(ii) Write down the final states set(s) of $\mathcal{A}$ explicitly.

$F_1 = \{p_0q_0, p_0q_1, p_0q_2, p_2q_0, p_2q_1, p_2q_2\}$, $F_2 = \{p_1q_0, p_1q_1, p_1q_2\}$, and $F_3 = \{p_0q_0, p_1q_0, p_2q_0, p_0q_1, p_1q_1, p_2q_1\}$.

(iii) Is $\mathcal{L}(\mathcal{A}) = \varnothing$? If not, provide a word which is contained in $\mathcal{L}(\mathcal{A})$.

$(\text{a a b})^\omega \in \mathcal{L}(\mathcal{A})$.

Exercise 2 (20 points)

Consider the following nanoPromela statement.

```
do
  :: a  => if :: d => e ! f fi ; g ! h
  :: b  => i ! j ; if :: k => l ! m fi
  :: c  => skip
od
```

Formally derive all transitions that are possible from this initial statement. You may use abbreviations like "`do ... od`" and "`if ... fi`".

$$\frac{}{\texttt{do ... od} \xrightarrow{\neg a \wedge \neg b \wedge \neg c} \text{exit}}$$

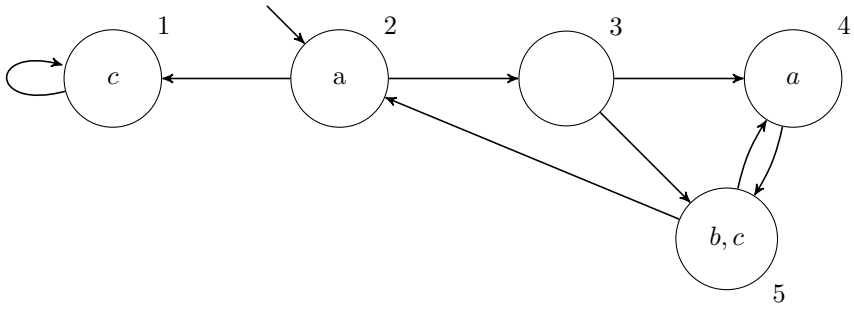
$$\frac{\dfrac{}{\text{skip} \longrightarrow \text{exit}}}{\texttt{do ... od} \xrightarrow{c} \texttt{do ... od}}$$

$$\frac{\dfrac{\dfrac{}{\texttt{i ! j} \xrightarrow{\texttt{i ! j}} \text{exit}}}{\texttt{i ! j; if ... fi} \xrightarrow{\texttt{i ! j}} \texttt{if ... fi}}}{\texttt{do ... od} \xrightarrow{b \,:\, \texttt{i ! j}} \texttt{if ... fi; do ... od}}$$

$$\frac{\dfrac{\dfrac{\dfrac{}{\texttt{e ! f} \xrightarrow{\texttt{e ! f}} \text{exit}}}{\texttt{if ... fi} \xrightarrow{d \,:\, \texttt{e ! f}} \text{exit}}}{\texttt{if ... fi; g ! h} \xrightarrow{d \,:\, \texttt{e ! f}} \texttt{g ! h}}}{\texttt{do ... od} \xrightarrow{a \wedge d \,:\, \texttt{e ! f}} \texttt{g ! h; do ... od}}$$

Exercise 3 (20 points)

Consider the following transition system TS.



Perform CTL*-model checking for the formula

$$\Phi = (\mathsf{A}\,((\mathsf{E}\,\mathsf{X}\,a) \Rightarrow \mathsf{X}\,a)) \wedge (\mathsf{E}\,(\neg a \wedge \neg b)\,\mathsf{U}\,c)$$

Here, the sets $Sat(\Psi)$ should be indicated for every non-atomic state-subformula Ψ of Φ. Note that the subformula $\neg a \wedge \neg b$ of Φ should be seen as a state-formula. It is not necessary to perform the LTL-model checking explicitly, but write down each LTL-formula that is checked.

- Eliminating A yields the formula $\Phi' = (\neg\mathsf{E}\,\neg((\mathsf{E}\,\mathsf{X}\,a) \Rightarrow \mathsf{X}\,a)) \wedge (\mathsf{E}\,(\neg a \wedge \neg b)\,\mathsf{U}\,c)$.
- $Sat(\neg a) = \{1, 3, 5\}$
- $Sat(\neg b) = \{1, 2, 3, 4\}$
- $Sat(\neg a \wedge \neg b = \Psi_1) = \{1, 3\}$
- $Sat(\mathsf{E}\,\Psi_1\,\mathsf{U}\,c = \Psi_2) = \{1, 3, 5\}$ (LTL model checking of formula $a_{\Psi_1}\,\mathsf{U}\,c$)
- $Sat(\mathsf{E}\,\mathsf{X}\,a = \Psi_3) = \{3, 5\}$ (LTL model checking of formula $\mathsf{X}\,a$)
- $Sat(\mathsf{E}\,\neg(\Psi_3 \Rightarrow \mathsf{X}\,a) = \Psi_4) = \{3\}$ (LTL model checking of formula $\neg(a_{\Psi_3} \Rightarrow \mathsf{X}\,a) \equiv a_{\Psi_3} \wedge \mathsf{X}\,\neg a$)
- $Sat(\neg\Psi_4 = \Psi_5) = \{1, 2, 3, 4, 5\} \setminus \{3\} = \{1, 2, 4, 5\}$
- $Sat(\Phi') = \{1, 3, 5\} \cap \{1, 2, 4, 5\} = \{1, 5\}$

$\Rightarrow$ $TS \not\models \Phi$

Exercise 4 (9 points)

Each correct answer is worth 3 points. A wrong answer results in zero points (for that question, not for the whole exercise). Giving no answer is worth 1 point.

	Yes	No
$\mathsf{F}\,(a\,\mathsf{U}\,b) \equiv \neg\mathsf{E}\,\mathsf{G}\,\neg b$.	✓	
The LTL formula $\mathsf{F}\,(a\,\mathsf{U}\,\mathsf{X}\,(b\,\mathsf{U}\,\mathsf{X}\,a))$ describes the following property: Every path contains two a's with a b in between.		✓
The number of states of the GNBA $\mathcal{A}_\varphi$ for some LTL-formula φ using the improved translation is $1 + 2^n$ where n is the number of temporal operators in φ.		✓