Introduction to Model Checking (VO) WS 2009/2010 LVA 703503

First name: ____________________

Last name: ____________________

Matriculation number: ____________________

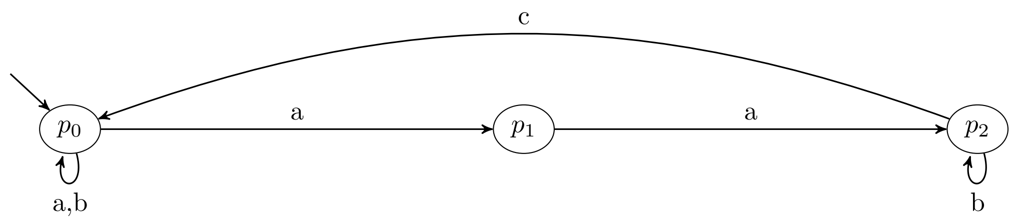
- Please answer all exercises in a readable and precise way.
- Please cross out solution attempts which are replaced by another solution.
- Please do not remove the staples of the exam.
- Cheating is not allowed. Everyone who is caught will fail the exam.

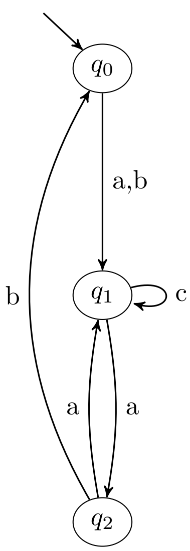
Exercise	**Maximal points**	**Points**
1	**20**	
2	**20**	
3	**21**	
4	**9**	
Σ	**70**	
Grade		

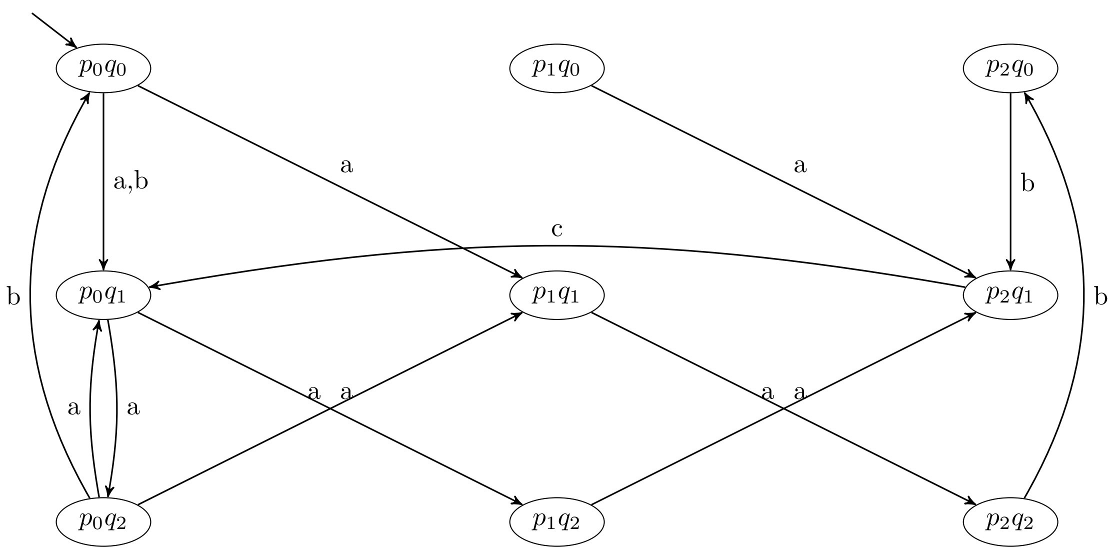
Exercise 1 (14 + 3 + 3 points)

Consider the GNBAs $\mathcal{A}_1 = (\{p_0, p_1, p_2\}, \Sigma, p_0, \delta_1, \{p_0, p_2\}, \{p_1\})$ and $\mathcal{A}_2 = (\{q_0, q_1, q_2\}, \Sigma, q_0, \delta_2, \{q_2\})$.

(i) Construct the GNBA $\mathcal{A}$ for the intersection of $\mathcal{A}_1$ and $\mathcal{A}_2$.







(ii) Write down the final states set(s) of $\mathcal{A}$ explicitly.

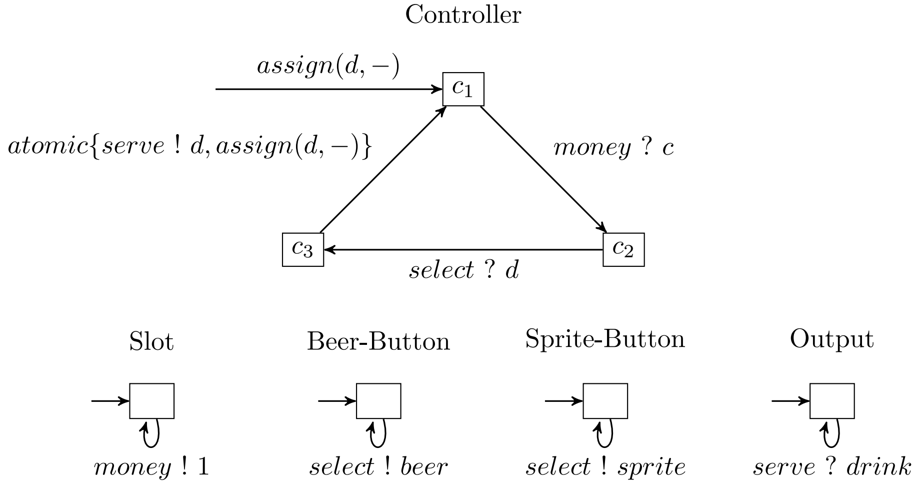
$F_1 = \{p_0q_0, p_0q_1, p_0q_2, p_2q_0, p_2q_1, p_2q_2\}$, $F_2 = \{p_1q_0, p_1q_1, p_1q_2\}$, and $F_3 = \{p_0q_2, p_1q_2, p_2q_2\}$.

(iii) Is $\mathcal{L}(\mathcal{A}) = \varnothing$? If not, provide a word which is contained in $\mathcal{L}(\mathcal{A})$.

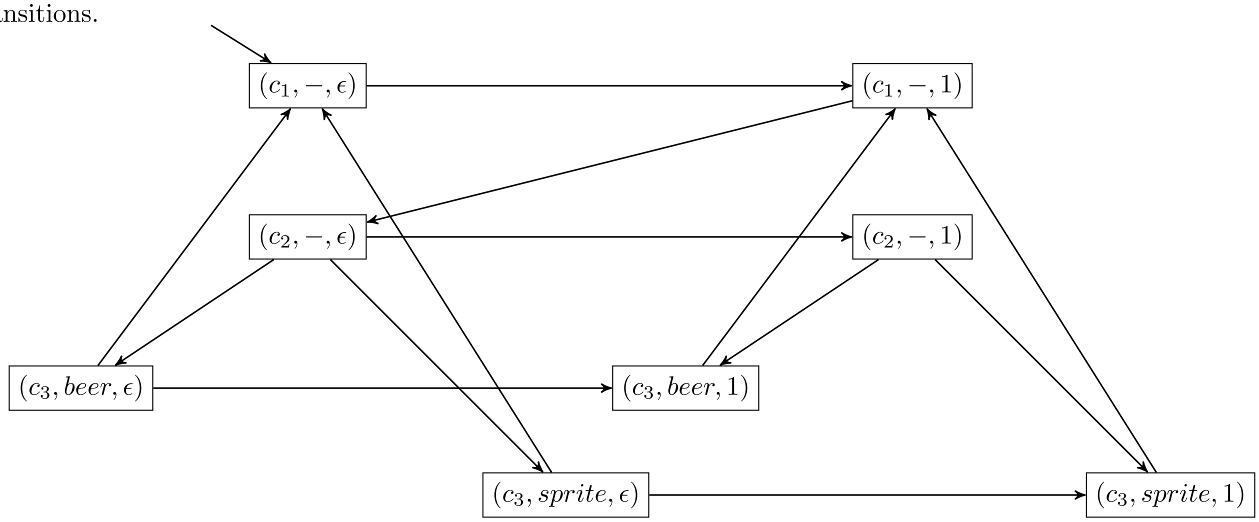
(a a b b c a b)$^\omega \in \mathcal{L}(\mathcal{A})$.

Exercise 2 (20 points)

Consider the following channel system [Slot | Beer-Button | Sprite-Button | Controller | Output] which models a distributed beverage vending machine. Here, communication is done via three channels where the capacity of the *money*-channel is 1, and the capacity of the *select*- and *serve*-channel is 0.



Complete the following transition system where a state (c_i, x, c) represents the current location in the controller c_i, the value x of the variable d of the controller, and the value c of the *money*-channel. You do not have to label transitions.



Exercise 3 (6 + 15 points)

Consider the following formula:

$$\varphi = \neg(\mathsf{true}\,\mathsf{U}\,(\mathsf{red} \wedge \neg(\neg\mathsf{green} \wedge \mathsf{X}\,(\neg\mathsf{green}\,\mathsf{U}\,\neg\mathsf{red})))$$

The following exercises can be done independently!

(i) Construct a simplified formula ψ with $\varphi \equiv \psi$ by introducing operators like F, G, $\vee$, $\Rightarrow$, ... Then try to formulate the meaning of ψ in words (German or English).

$$\begin{aligned}\varphi &\equiv \neg(\mathsf{F}\,(\mathsf{red} \wedge \neg(\neg\mathsf{green} \wedge \mathsf{X}\,(\neg\mathsf{green}\,\mathsf{U}\,\neg\mathsf{red})))\\ &\equiv \mathsf{G}\,\neg(\mathsf{red} \wedge \neg(\neg\mathsf{green} \wedge \mathsf{X}\,(\neg\mathsf{green}\,\mathsf{U}\,\neg\mathsf{red})))\\ &\equiv \mathsf{G}\,(\mathsf{red} \Rightarrow (\neg\mathsf{green} \wedge \mathsf{X}\,(\neg\mathsf{green}\,\mathsf{U}\,\neg\mathsf{red})))\\ &\equiv \mathsf{G}\,(\mathsf{red} \Rightarrow (\neg\mathsf{green} \wedge \mathsf{F}\,\neg\mathsf{red}))\\ &\equiv \mathsf{G}\,(\neg\mathsf{red} \vee \neg\mathsf{green}) \wedge \mathsf{G}\,\mathsf{F}\,\neg\mathsf{red}\end{aligned}$$

Green and red do not occur at the same time and infinitely often it is not red.

(ii) Construct the automaton for φ using the improved translation.
$\mathcal{A}_\varphi = (\{q_0\} \uplus 2^5, 2^2, q_0, \delta, F_1, F_2)$ where

- The reduced Fischer Ladner closure is

$$cl'(\varphi) = \mathsf{red}, \mathsf{green}, \underbrace{\neg\mathsf{green}\,\mathsf{U}\,\neg\mathsf{red}}_{\varphi_3}, \underbrace{\mathsf{X}\,\varphi_3}_{\varphi_4}, \underbrace{\mathsf{true}\,\mathsf{U}\,(\mathsf{red} \wedge \neg(\neg\mathsf{green} \wedge \varphi_4))}_{\varphi_5},$$

- $(c_1, \ldots, c_5)^T \in \delta(q_0, (d_1, d_2)^T)$ iff $(c_1 \Leftrightarrow d_1) \wedge (c_2 \Leftrightarrow d_2) \wedge \neg c_5$

- $(c_1, \ldots, c_5)^T \in \delta((b_1, \ldots, b_5)^T, (d_1, d_2)^T)$ iff $(c_1 \Leftrightarrow d_1) \wedge (c_2 \Leftrightarrow d_2) \wedge (b_3 \Leftrightarrow (\neg b_1 \vee (\neg b_2 \wedge c_3))) \wedge (b_4 \Leftrightarrow c_3) \wedge (b_5 \Leftrightarrow ((b_1 \wedge \neg(\neg b_2 \wedge b_4)) \vee c_5))$

- $F_1 = \{(b_1, \ldots, b_5)^T \mid \neg b_3 \vee \neg b_1\}$
  $F_2 = \{(b_1, \ldots, b_5)^T \mid \neg b_5 \vee (b_1 \wedge \neg(\neg b_2 \wedge b_4))\}$

Compute the number of direct preceding states of state $(1, 0, 0, 1, 1)^T$. Simplifying the conditions yields $b_3 \Leftrightarrow \neg b_1$, $b_4 = 0$, $b_5 = 1$ and hence, there are the 4 states of the form $(b_1, *, \neg b_1, 0, 1)$. (Obviously, q_0 is not a preceding state, as it would require the last entry to be 0.)

Exercise 4 (9 points)

Give an algorithm which decides for two LTL formulas φ and ψ whether $\mathcal{L}(\varphi) = \mathcal{L}(\psi)$. Prove the correctness.

Algorithm: Output "$\mathcal{L}(\neg(\varphi \Leftrightarrow \psi)) = \varnothing$".

- the output is computable as we can construct the GNBA for $\neg(\varphi \Leftrightarrow \psi)$ and then decide emptyness of that GNBA.
- the algorithm is sound since

$$\begin{array}{ll} & \mathcal{L}(\varphi) = \{w \mid w \models \varphi\} = \{w \mid w \models \psi\} = \mathcal{L}(\psi) \\ \text{iff} & \mathcal{L}(\varphi \Leftrightarrow \psi) = \{w \mid w \models \varphi \Leftrightarrow \psi\} = \Sigma^\omega \\ \text{iff} & \mathcal{L}(\neg(\varphi \Leftrightarrow \psi)) = \{w \mid w \models \neg(\varphi \Leftrightarrow \psi)\} = \varnothing \end{array}$$