University of Innsbruck — Institute of Computer Science
3rd exam — June 15, 2010

Introduction to Model Checking (VO) — WS 2009/2010 — LVA 703503

First name: ______________________

Last name: ______________________

Matriculation number: ______________________

- Please answer all exercises in a readable and precise way.
- Please cross out solution attempts which are replaced by another solution.
- Please do not remove the staples of the exam.
- Cheating is not allowed. Everyone who is caught will fail the exam.

Exercise	**Maximal points**	**Points**
1	**22**	
2	**21**	
3	**21**	
4	**6**	
Σ	**70**	
Grade		

Exercise 1 (11 + 11 points)

Consider the following properties over $AP = \{s, t\}$ or $\Sigma = 2^{AP}$.

- P_1: There are infinitely many s's
- P_2: The input is of the form $(\{s\}\{s\}\{s,t\})^\omega$
- P_3: If P_1 holds then P_2 holds.

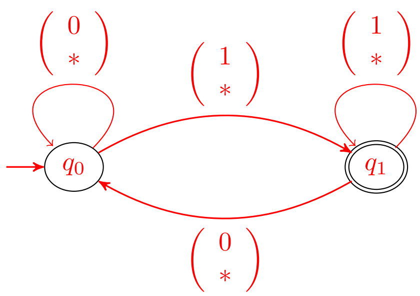
Specify each of the properties P_i

(i) as LTL formula φ_i

(ii) as NBA $\mathcal{A}_i$

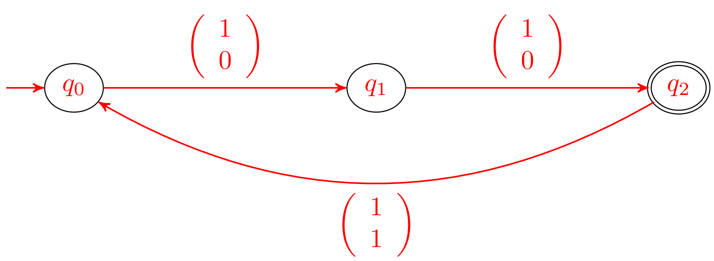
(i)
- $\varphi_1 = \mathsf{G}\,\mathsf{F}\,s$
- $\varphi_2 = \mathsf{G}\,s \wedge \neg t \wedge \mathsf{X}\,\neg t \wedge \mathsf{X}\,\mathsf{X}\,t \wedge \mathsf{G}\,(t \Rightarrow (\mathsf{X}\,\neg t \wedge \mathsf{X}\,\mathsf{X}\,\neg t \wedge \mathsf{X}\,\mathsf{X}\,\mathsf{X}\,t))$
- $\varphi_3 = \varphi_1 \Rightarrow \varphi_2$

$\mathcal{A}_1$:



(ii)
-

$\mathcal{A}_2$:



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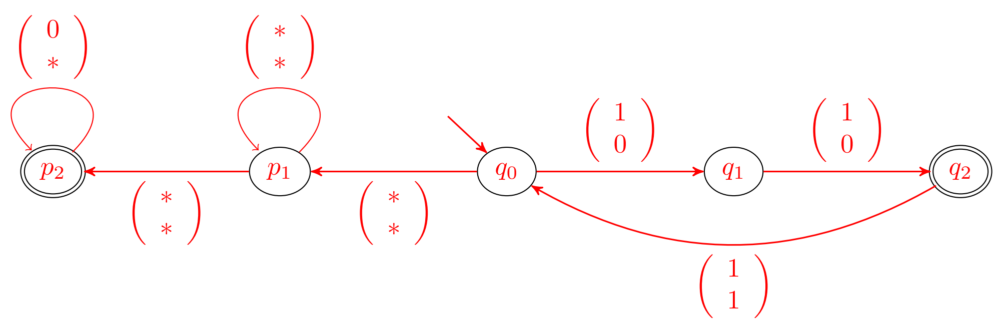
$\mathcal{A}_3$:



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Exercise 2 (21 points)

Consider the following nanoPromela statement.

```
do
  :: x=1  => if :: y=2  => c ! 3
                :: y=4  => skip
             fi ;
             d ? z
od
```

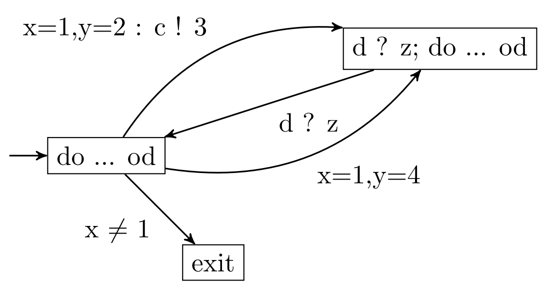
Construct the program graph for this statement. For the whole statement, additionally derive all transitions formally using the inference rules.

You may use abbreviations like "`do ... od`", "`if ... fi`", . . .

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\text{c ! 3} \xrightarrow{\text{c ! 3}} \text{exit}}}{\text{if ... fi} \xrightarrow{\text{y = 2 : c ! 3}} \text{exit}}}{\text{if ... fi; d ? z} \xrightarrow{\text{y = 2 : c ! 3}} \text{d ? z}}}{\text{do ... od} \xrightarrow{\text{x = 1, y = 2 : c ! 3}} \text{d ? z; do ... od}}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\text{skip} \longrightarrow \text{exit}}}{\text{if ... fi} \xrightarrow{\text{y = 4}} \text{exit}}}{\text{if ... fi; d ? z} \xrightarrow{\text{y = 4}} \text{d ? z}}}{\text{do ... od} \xrightarrow{\text{x = 1, y = 4}} \text{d ? z; do ... od}}$$

$$\dfrac{}{\text{do ... od} \xrightarrow{\text{x} \neq 1} \text{exit}}$$



Exercise 3 (7 + 14 points)

Consider the following formula:

$$\varphi = (\mathsf{a}\,\mathsf{U}\,\mathsf{b})\,\mathsf{U}\,(\mathsf{X}\,\mathsf{a})$$

(i) Fill in all values of the φ-expansion for the given input word that can uniquely be determined where . . . may be arbitrary.

a	0	1	0	0	1	0	0	0	0	. . .
b	0	1	1	1	0	1	0	1	1	. . .
X a	1	0	0	1	0	0	0	0	–	
a U b	0	1	1	1	1	1	0	1	1	
φ	1	1	1	1	0	0	0	–	–	

(ii) Construct the automaton for φ.

$\mathcal{A}_\varphi = (\{q_0\} \uplus 2^5, 2^2, q_0, \delta, F_1, F_2)$ where the Fischer Ladner closure is $cl(\varphi) = \mathsf{a}, \mathsf{b}, \underbrace{\mathsf{X}\,\mathsf{a}}_{\varphi_3}, \underbrace{\mathsf{a}\,\mathsf{U}\,\mathsf{b}}_{\varphi_4}, \varphi$

- $(c_1, \ldots, c_5)^T \in \delta(q_0, (d_1, d_2)^T)$ iff $(c_1 \Leftrightarrow d_1) \wedge (c_2 \Leftrightarrow d_2) \wedge c_5$

- $(c_1, \ldots, c_5)^T \in \delta((b_1, \ldots, b_5)^T, (d_1, d_2)^T)$ iff $(c_1 \Leftrightarrow d_1) \wedge (c_2 \Leftrightarrow d_2) \wedge (b_3 \Leftrightarrow c_1) \wedge (b_4 \Leftrightarrow b_2 \vee (b_1 \wedge c_4)) \wedge (b_5 \Leftrightarrow b_3 \vee (b_4 \wedge c_5))$

- $F_1 = \{(b_1, \ldots, b_5)^T \mid \neg b_4 \vee b_2\}$
  $F_2 = \{(b_1, \ldots, b_5)^T \mid \neg b_5 \vee b_3\}$

Explicitly list all transitions leading to state $q = (1, 0, 0, 1, 1)^T$. Simplifying the conditions yields $d_1 \wedge \neg d_2 \wedge b_3 \wedge (b_4 \Leftrightarrow b_2 \vee b_1) \wedge b_5$ and hence,

$$\begin{array}{rcl}
(0,0,1,0,1)^T & \xrightarrow{(1,0)^T} & q \\
(0,1,1,1,1)^T & \xrightarrow{(1,0)^T} & q \\
(1,0,1,1,1)^T & \xrightarrow{(1,0)^T} & q \\
(1,1,1,1,1)^T & \xrightarrow{(1,0)^T} & q \\
q_0 & \xrightarrow{(1,0)^T} & q
\end{array}$$

Exercise 4 (6 points)

Each correct answer is worth 2 points. For each wrong answer 1 point is subtracted. It is not possible to get negative points from this exercise.

	Yes	No
LTL model checking is PSPACE complete.	✓	
Consider restricted nanoPromela programs which may have at most 2 boolean variables (and no other variables). Then the size of the resulting transition system is polynomial in the size of the program.		✓
Satisfiablity of LTL-formulas is decidable.	✓	