1. Consider the following propositional formulas

$$\begin{array}{ll} A & (\mathsf{p} \to \neg \mathsf{q}) \to (\mathsf{q} \lor \neg \mathsf{p}) \\ B & ((\mathsf{q} \to (\mathsf{p} \lor (\mathsf{q} \to \mathsf{p}))) \lor \neg(\mathsf{p} \to \mathsf{q})) \to \mathsf{p} \\ C & (\mathsf{p} \to \mathsf{q}) \to ((\mathsf{r} \to \mathsf{s}) \to (\mathsf{p} \land \mathsf{r} \to \mathsf{q} \land \mathsf{s})) \end{array}$$

   a) Show that formula A is neither a tautology, nor contradictory, by providing suiable assignments. (2 pts)

   b) Show that formula B is neither a tautology, nor contradictory, by providing suiable assignments. (2 pts)

   c) Show that formula C is valid by providing a formal natural deduction or resolution proof. (4 pts)

2. Consider first-order logic and the following attempt at defining resolution, where we assume the definition of *resolution* and *factoring* from the lecture.

   – $\mathsf{Res}_1(\mathcal{C}) := \{D \mid D$ conclusion of resolution/factoring with premises in $\mathcal{C}\} \cup \mathcal{C}$,

   – $\mathsf{Res}_1^0(\mathcal{C}) := \mathcal{C}$, $\mathsf{Res}_1^{n+1}(\mathcal{C}) := \mathsf{Res}_1(\mathsf{Res}_1^n(\mathcal{C}))$, and $\mathsf{Res}_1^*(\mathcal{C}) := \bigcup_{n \geqslant 0} \mathsf{Res}_1^n(\mathcal{C})$.

   a) Compare the definition of the operator Res_1^* with the definition of the operator Res^* as defined in the lecture. (2 pts)

   b) Is the definition of Res_1^* correct, i.e., do we have $\Box \in \mathsf{Res}^*(\mathcal{C})$ iff $\Box \in \mathsf{Res}_1^*(\mathcal{C})$ for any clause set $\mathcal{C}$? Explain your answer. (4 pts)

   c) Consider the orignal definition of the resolution operator, but generalise *positive factoring* to factoring of positive or negative atomic formulas. Call the resulting resolution operator Res_2. Is the following statement correct: $\Box \in \mathsf{Res}^*(\mathcal{C})$ iff $\Box \in \mathsf{Res}_2^*(\mathcal{C})$ for any clause set $\mathcal{C}$? Explain your answer. (4 pts)

3. Consider the following sentences:

   ① Some students are older than some instructor.

   ② It is not the case that all students are younger than some instructor.

   ③ Some students and some instructors have children.

   ④ Some instructors have children that are students.

   ⑤ There exists a male instructor, whose child is called Lena.

   a) For each of the sentences above, give a first-order formula that formalises the sentence. Use therefore *at most* the following constants and predicates:

      – Individual constants: andy, eva, lena, paul.

- Predicates constants: Student, Instructor, Female, Male, which are unary and Older, Mother, Father, which are binary.

The interpretation of the unary predicates follows their names, $\mathsf{Older}(x, y)$ represents that "x is a older than y", while $\mathsf{Mother}(x, y)/\mathsf{Father}(x, y)$ means "x is mother/father of y". (5 pts)

b) Show that your formalisation is satisfiable. (3 pts)

c) Sentences ① and ② appear equivalent, is this really true? Explain your answer. (3 pts)

4. Consider the following first-order formulas with predicate constants P, Q, and R:

$$C := \forall x \exists y \forall z \forall u \exists w\, (\mathsf{Q}(x, y, z) \to \mathsf{P}(w, x, y, u))$$
$$D := \exists x \forall y \forall z \exists w\, (\mathsf{R}(x, z) \wedge \mathsf{R}(x, y) \to \mathsf{R}(x, w) \wedge \mathsf{R}(y, w) \wedge \mathsf{R}(z, w))$$
$$E := \forall x(\neg \mathsf{Q}(x) \to \mathsf{R}(x)) \to \neg(\forall x \neg \mathsf{R}(x) \wedge \exists x \neg \mathsf{Q}(x))$$

a) Give the SNF of formula C. (3 pts)

b) Give the SNF of formula D. (3 pts)

a) Use resolution for first-order to show that formula E is valid. (5 pts)

5. Determine whether the following statements are true or false. Give your answers on the answer sheet. Every correct answer is worth 1 points and *every wrong -1 points.* (10 pts)

   - Consider propositional logic. Then $A_1, \ldots, A_n \models B$, asserts that $\mathsf{v}(B) = \mathsf{T}$, whenever there exists $i \in \{1, \ldots, n\}$ such that $\mathsf{v}(A_i) = \mathsf{T}$, for any assignment v.
   - Natural deduction for propositional logic is sound and complete. Furthermore it is the only formal system with these properties.
   - Let $\mathcal{A}$, $\mathcal{B}$ be first-order structures such that $\mathcal{A} \cong \mathcal{B}$. Then for every sentence F we have $\mathcal{A} \models F$ iff $\mathcal{B} \models F$.
   - If every finite subset of a set of first-order formulas $\mathcal{G}$ has a countable model, then $\mathcal{G}$ has a countable model.
   - Suppose $\mathcal{G}$ is a set of first-order formulas and $\mathcal{G} \vdash F$. Then there exists a finite subset $\mathcal{G}_0 \subseteq \mathcal{G}$ such that $\mathcal{G}_0 \vdash F$.
   - Let S be the set of satisfiable sets of first-order formulas $\mathcal{G}$. Then S fulfils the satisfaction properties.
   - Let $\mathcal{G}$ be a set of first-order formulas and let F be a first-order formula such that $\mathcal{G} \vdash F$. Then $\mathcal{G} \models \neg F$.
   - There exists a satisfiable and universal first-order sentence (without $=$) F, such that F doesn't have a Herbrand model.
   - A unifier σ of expressions E and F is a ground substitution such that $E\sigma = F\sigma$.
   - For any first-order sentence F there exists a set of clauses $\mathcal{C} = \{C_1, \ldots, C_m\}$ such that $F \approx \forall x_1 \ldots \forall x_n(C_1 \wedge \cdots \wedge C_m)$.