1. a) The assignment $\mathsf{v}(\mathsf{p}) = \mathsf{v}(\mathsf{q}) = \mathsf{T}$ suffices to show that A is satisfiable; the assignment $\mathsf{v}(\mathsf{p}) = \mathsf{T}$ and $\mathsf{v}(\mathsf{q}) = \mathsf{F}$, yields that B is not a tautology.

   b) Any assignment that satisfies $\mathsf{v}(\mathsf{p}) = \mathsf{T}$ suffices to show that B is satisfiable; the assignment $\mathsf{v}(\mathsf{p}) = \mathsf{v}(\mathsf{q}) = \mathsf{F}$, yields that B is not a tautology.

   c) We show that the formula $\neg C$ is inconsistent. Transformation of $\neg C$ yields the following clause set:

$$\mathsf{p} \quad \mathsf{r} \quad \neg\mathsf{p} \vee \mathsf{q} \quad \neg\mathsf{r} \vee \mathsf{s} \quad \neg\mathsf{q} \vee \neg\mathsf{s}$$

   The following resolution proof shows that formula $\neg C$ is unsatisfiable:

$$\dfrac{\dfrac{\neg\mathsf{p} \vee \mathsf{q} \quad \mathsf{p}}{\mathsf{q}} \qquad \dfrac{\dfrac{\neg\mathsf{r} \vee \mathsf{s} \quad \mathsf{r}}{\mathsf{s}} \quad \neg\mathsf{q} \vee \neg\mathsf{s}}{\neg\mathsf{q}}}{\square}$$

2. a) See Definition 8.4 on page 58.

   b) The definition of Res_1^* is correct, as the resolution operator is now monotone.

   c) The definition of Res_2 is correct, as unrestricted factoring is more general than positive factoring.

3. a) ① $\exists x \exists y (\mathsf{Student}(x) \wedge \mathsf{Instructor}(x) \wedge \mathsf{Older}(x, y))$

   ② $\neg \forall x (\mathsf{Student}(x) \rightarrow (\exists y (\mathsf{Instructor}(y) \wedge \mathsf{Older}(y, x))))$

   ③ $\exists x \exists y \exists v \exists w (\mathsf{Student}(x) \wedge \mathsf{Instructor}(v) \wedge (\mathsf{Mother}(x, y) \vee \mathsf{Father}(x, y)) \wedge (\mathsf{Mother}(v, w) \vee \mathsf{Father}(v, w)))$

   ④ $\exists x \exists y (\mathsf{Instructor}(x) \wedge (\mathsf{Mother}(x, y) \vee \mathsf{Father}(x, y)) \wedge \mathsf{Student}(y))$

   ⑤ $\exists x (\mathsf{Instructor}(x) \wedge \mathsf{Male}(x) \wedge \mathsf{Father}(x, \mathsf{lena}))$

   b) We define a suitable structure $\mathcal{A} = (A, a)$ where $A = \{a, e, l, p\}$ and the mapping a is defined as follows:

   - $a(\mathsf{andy}) = a$, $a(\mathsf{eva}) = e$, $a(\mathsf{lena}) = l$, $a(\mathsf{paul}) = p$.
   - $a(\mathsf{Student}) = \{a\}$, $a(\mathsf{Instructor}) = \{p\}$, $a(\mathsf{Older}) = \{(a, p)\}$, $a(\mathsf{Father}) = \{(p, l), (a, e)\}$, and the interpretation of all other predicates is empty.

   Then $\mathcal{A} \models F$ holds for each of the five sentences above and thus the formalisation is satisfiable.

c) The formula representing Sentences ② is logically equivalent to $\exists x(\mathsf{Student}(x) \wedge \forall y(\neg\mathsf{Instructor}(y) \vee \neg\mathsf{Older}(y,x))$. This is not logically equivalent to the formula representing Sentences ①.

4. a) The SNF of formula C has the form

$$\forall x \forall z \forall u (\neg \mathsf{Q}(x, \mathsf{f}(x), z) \vee \mathsf{P}(\mathsf{g}(x, z, u), x, \mathsf{f}(x), u)) \ ,$$

where f, g are new Skolem functions.

b) The SNF of formula D has the form

$$\begin{aligned} \forall y \forall z \, ((\neg \mathsf{R}(\mathsf{a}, z) \vee \neg \mathsf{R}(\mathsf{a}, y) \vee \mathsf{R}(\mathsf{a}, \mathsf{f}(y, z))) \wedge \\ (\neg \mathsf{R}(\mathsf{a}, z) \vee \neg \mathsf{R}(\mathsf{a}, y) \vee \mathsf{R}(y, \mathsf{f}(y, z))) \wedge \\ (\neg \mathsf{R}(\mathsf{a}, z) \vee \neg \mathsf{R}(\mathsf{a}, y) \vee \mathsf{R}(z, \mathsf{f}(y, z)))) \ , \end{aligned}$$

where a is a new Skolem constants, f a new Skolem function.

(a) First we negate E and transform the result to obtain for example the following corresponding SNF:

$$\forall x \forall y \, ((\mathsf{R}(x) \vee \mathsf{Q}(x)) \wedge \neg \mathsf{R}(y) \wedge \neg \mathsf{Q}(\mathsf{a})) \ ,$$

where a is a new Skolem constant. We obtain $\mathcal{C} = \{\mathsf{R}(x) \vee \mathsf{Q}(x), \neg \mathsf{R}(y), \neg \mathsf{Q}(\mathsf{a})\}$ as the corresponding set of clauses. A possible resolution proof is given below, where for each inference σ denotes the most general unifier.

$$\dfrac{\dfrac{\mathsf{R}(x) \vee \mathsf{Q}(x) \quad \neg \mathsf{R}(y)}{\mathsf{Q}(x)} \ \sigma = \{y \mapsto x\} \qquad \neg \mathsf{Q}(\mathsf{a})}{\Box} \ \sigma = \{x \mapsto \mathsf{a}\}$$

5.

statement	**yes**	**no**
Consider propositional logic. Then $A_1, \ldots, A_n \models B$, asserts that $\mathsf{v}(B) = \mathsf{T}$, whenever there exists $i \in \{1, \ldots, n\}$ such that $\mathsf{v}(A_i) = \mathsf{T}$, for any assignment v.	☐	☑
Natural deduction for propositional logic is sound and complete. Furthermore it is the only formal system with these properties.	☐	☑
Let $\mathcal{A}$, $\mathcal{B}$ be first-order structures such that $\mathcal{A} \cong \mathcal{B}$. Then for every sentence F we have $\mathcal{A} \models F$ iff $\mathcal{B} \models F$.	☑	☐
If every finite subset of a set of first-order formulas $\mathcal{G}$ has a countable model, then $\mathcal{G}$ has a countable model.	☑	☐
Suppose $\mathcal{G}$ is a set of first-order formulas and $\mathcal{G} \vdash F$. Then there exists a finite subset $\mathcal{G}_0 \subseteq \mathcal{G}$ such that $\mathcal{G}_0 \vdash F$.	☑	☐
Let S be the set of satisfiable sets of first-order formulas $\mathcal{G}$. Then S fulfils the satisfaction properties.	☑	☐
Let $\mathcal{G}$ be a set of first-order formulas and let F be a first-order formula such that $\mathcal{G} \vdash F$. Then $\mathcal{G} \models \neg F$.	☐	☑
There exists a satisfiable and universal first-order sentence F (without $=$), such that F doesn't have a Herbrand model.	☐	☑
A unifier σ of expressions E and F is a ground substitution such that $E\sigma = F\sigma$.	☐	☑
For any first-order sentence F there exists a set of clauses $\mathcal{C} = \{C_1, \ldots, C_m\}$ such that $F \approx \forall x_1 \ldots \forall x_n (C_1 \land \cdots \land C_m)$.	☑	☐